Chapter 9

Rational Expressions and Equations

What You'll Learn

- **Lessons 9-1 and 9-2** Simplify rational expressions.
- **Lesson 9-3** Graph rational functions.
- **Lesson 9-4** Solve direct, joint, and inverse variation problems.
- **Lesson 9-5** Identify graphs and equations as different types of functions.
- **Lesson 9-6** Solve rational equations and inequalities.

Key Vocabulary

- **rational expression** (p. 472)
- **asymptote** (p. 485)
- **point discontinuity** (p. 485)
- **direct variation** (p. 492)
- **inverse variation** (p. 493)

Why It's Important

Rational expressions, functions, and equations can be used to solve problems involving mixtures, photography, electricity, medicine, and travel, to name a few. Direct, joint, and inverse variation are important applications of rational expressions. For example, scuba divers can use direct variation to determine the amount of pressure at various depths. *You will learn how to determine the amount of pressure exerted on the ears of a diver in Lesson 9-4.*

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 9.

For Lesson 9-1 — Solve Equations with Rational Numbers

Solve each equation. Write your answer in simplest form. *(For review, see Lesson 1-3.)*

1. $\frac{8}{5}x = \frac{4}{15}$
2. $\frac{27}{14}t = \frac{6}{7}$
3. $\frac{3}{10} = \frac{12}{25}a$

4. $\frac{6}{7} = 9m$
5. $\frac{9}{8}b = 18$
6. $\frac{6}{7}s = \frac{3}{4}$

7. $\frac{1}{3}r = \frac{5}{6}$
8. $\frac{2}{3}n = 7$
9. $\frac{4}{5}r = \frac{5}{6}$

For Lesson 9-3 — Determine Asymptotes and Graph Equations

Draw the asymptotes and graph each hyperbola. *(For review, see Lesson 8-5.)*

10. $\frac{(x-3)^2}{4} - \frac{(y+5)^2}{9} = 1$
11. $\frac{y^2}{4} - \frac{(x+4)^2}{1} = 1$
12. $\frac{(x+2)}{4} - \frac{(y-3)^2}{25} = 1$

For Lesson 9-4 — Solve Proportions

Solve each proportion.

13. $\frac{3}{4} = \frac{r}{16}$
14. $\frac{8}{16} = \frac{5}{y}$
15. $\frac{6}{8} = \frac{m}{20}$

16. $\frac{t}{3} = \frac{5}{24}$
17. $\frac{5}{a} = \frac{6}{18}$
18. $\frac{3}{4} = \frac{b}{6}$

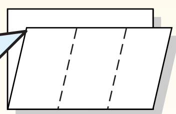
19. $\frac{v}{9} = \frac{12}{18}$
20. $\frac{7}{p} = \frac{1}{4}$
21. $\frac{2}{5} = \frac{3}{z}$

FOLDABLES™ Study Organizer

Make this Foldable to help you organize what you learn about rational expressions and equations. Begin with a sheet of plain $8\frac{1}{2}$" × 11" paper.

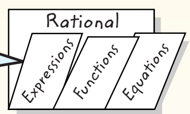
Step 1 Fold

Fold in half lengthwise leaving a $1\frac{1}{2}$" margin at the top. Fold again in thirds.



Step 2 Cut and Label

Open. Cut along the second folds to make three tabs. Label as shown.



Reading and Writing As you read and study the chapter, write notes and examples for each concept under the tabs.

CONTENTS

9-1 Multiplying and Dividing Rational Expressions

What You'll Learn

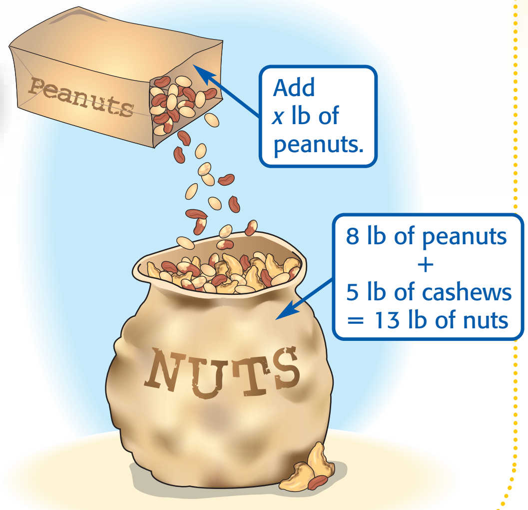
- Simplify rational expressions.
- Simplify complex fractions.

Vocabulary

- rational expression
- complex fraction

How are rational expressions used in mixtures?

The Goodie Shoppe sells candy and nuts by the pound. One of their items is a mixture of peanuts and cashews. This mixture is made with 8 pounds of peanuts and 5 pounds of cashews. Therefore, $\frac{8}{8+5}$ or $\frac{8}{13}$ of the mixture is peanuts. If the store manager adds an additional x pounds of peanuts to the mixture, then $\frac{8+x}{13+x}$ of the mixture will be peanuts.



SIMPLIFY RATIONAL EXPRESSIONS A ratio of two polynomial expressions such as $\frac{8+x}{13+x}$ is called a **rational expression**. Because variables in algebra represent real numbers, operations with rational numbers and rational expressions are similar.

To write a fraction in simplest form, you divide both the numerator and denominator by their greatest common factor (GCF). To simplify a rational expression, you use similar properties.

Example 1 Simplify a Rational Expression

a. Simplify $\frac{2x(x-5)}{(x-5)(x^2-1)}$.

Look for common factors.

$$\frac{2x(x-5)}{(x-5)(x^2-1)} = \frac{2x}{x^2-1} \cdot \frac{\overset{1}{\cancel{x-5}}}{\underset{1}{\cancel{x-5}}}$$ How is this similar to simplifying $\frac{10}{15}$?

$$= \frac{2x}{x^2-1}$$ Simplify.

b. Under what conditions is this expression undefined?

Just as with a fraction, a rational expression is undefined if the denominator is equal to 0. To find when this expression is undefined, completely factor the original denominator.

$$\frac{2x(x-5)}{(x-5)(x^2-1)} = \frac{2x(x-5)}{(x-5)(x-1)(x+1)}$$ $x^2 - 1 = (x-1)(x+1)$

The values that would make the denominator equal to 0 are 5, 1, or −1. So the expression is undefined when $x = 5$, $x = 1$, or $x = -1$. These numbers are called *excluded values*.

Standardized Test Practice

Example 2 Use the Process of Elimination

Multiple-Choice Test Item

For what value(s) of x is $\frac{x^2 + x - 12}{x^2 + 7x + 12}$ undefined?

(A) $-4, -3$ (B) -4 (C) 0 (D) $-4, 3$

Read the Test Item

You want to determine which values of x make the denominator equal to 0.

Solve the Test Item

Look at the possible answers. Notice that if x equals 0 or a positive number, $x^2 + 7x + 12$ must be greater than 0. Therefore, you can eliminate choices C and D. Since both choices A and B contain -4, determine whether the denominator equals 0 when $x = -3$.

$x^2 + 7x + 12 = (-3)^2 + 7(-3) + 12$ $x = -3$

$= 9 - 21 + 12$ Multiply.

$= 0$ Simplify.

Since the denominator equals 0 when $x = -3$, the answer is A.

The Princeton Review Test-Taking Tip

Sometimes you can save time by looking at the possible answers and eliminating choices, rather than actually evaluating an expression or solving an equation.

Sometimes you can factor out -1 in the numerator or denominator to help simplify rational expressions.

Example 3 Simplify by Factoring Out –1

Simplify $\frac{z^2w - z^2}{z^3 - z^3w}$.

$\frac{z^2w - z^2}{z^3 - z^3w} = \frac{z^2(w - 1)}{z^3(1 - w)}$ Factor the numerator and the denominator.

$= \frac{\overset{1}{\cancel{z^2}}(-1)\overset{1}{\cancel{(1 - w)}}}{\underset{z}{\cancel{z^3}}\underset{1}{\cancel{(1 - w)}}}$ $w - 1 = -(-w + 1)$ or $-1(1 - w)$

$= \frac{-1}{z}$ or $-\frac{1}{z}$ Simplify.

Remember that to multiply two fractions, you first multiply the numerators and then multiply the denominators. To divide two fractions, you multiply by the multiplicative inverse, or reciprocal, of the divisor.

Multiplication

$\frac{5}{6} \cdot \frac{4}{15} = \frac{\overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{2}} \cdot 2}{\underset{1}{\cancel{2}} \cdot 3 \cdot 3 \cdot \underset{1}{\cancel{5}}}$

$= \frac{2}{3 \cdot 3}$ or $\frac{2}{9}$

Division

$\frac{3}{7} \div \frac{9}{14} = \frac{3}{7} \cdot \frac{14}{9}$

$= \frac{\overset{1}{\cancel{3}} \cdot 2 \cdot \overset{1}{\cancel{7}}}{\underset{1}{\cancel{7}} \cdot \underset{1}{\cancel{3}} \cdot 3}$

$= \frac{2}{3}$

The same procedures are used for multiplying and dividing rational expressions.

Key Concept — Rational Expressions

Multiplying Rational Expressions

- **Words** To multiply two rational expressions, multiply the numerators and the denominators.
- **Symbols** For all rational expressions $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$, if $b \neq 0$ and $d \neq 0$.

Dividing Rational Expressions

- **Words** To divide two rational expressions, multiply by the reciprocal of the divisor.
- **Symbols** For all rational expressions $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$, if $b \neq 0$, $c \neq 0$, and $d \neq 0$.

The following examples show how these rules are used with rational expressions.

Study Tip

Alternative Method When multiplying rational expressions, you can multiply first and then divide by the common factors. For instance, in Example 4,

$\frac{4a}{5b} \cdot \frac{15b^2}{16a^3} = \frac{60ab^2}{80a^3b}$.

Now divide the numerator and denominator by the common factors.

$\frac{\overset{3}{\cancel{60}}\,\overset{1}{\cancel{a}}\,\overset{b}{\cancel{b^2}}}{\underset{4}{\cancel{80}}\,\underset{a^2}{\cancel{a^3}}\,\underset{1}{\cancel{b}}} = \frac{3b}{4a^2}$

Example 4 Multiply Rational Expressions

Simplify each expression.

a. $\frac{4a}{5b} \cdot \frac{15b^2}{16a^3}$

$$\frac{4a}{5b} \cdot \frac{15b^2}{16a^3} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{a}} \cdot 3 \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{b}} \cdot b}{\underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{b}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 2 \cdot 2 \cdot \underset{1}{\cancel{a}} \cdot a \cdot a}$$ Factor.

$$= \frac{3 \cdot b}{2 \cdot 2 \cdot a \cdot a}$$ Simplify.

$$= \frac{3b}{4a^2}$$ Simplify.

b. $\frac{8t^2s}{5r^2} \cdot \frac{15sr}{12t^3s^2}$

$$\frac{8t^2s}{5r^2} \cdot \frac{15sr}{12t^3s^2} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot \overset{1}{\cancel{t}} \cdot \overset{1}{\cancel{t}} \cdot \overset{1}{\cancel{s}} \cdot \overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{s}} \cdot \overset{1}{\cancel{r}}}{\underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{r}} \cdot r \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{t}} \cdot \underset{1}{\cancel{t}} \cdot t \cdot \underset{1}{\cancel{s}} \cdot \underset{1}{\cancel{s}}}$$ Factor.

$$= \frac{2}{rt}$$ Simplify.

Example 5 Divide Rational Expressions

Simplify $\frac{4x^2y}{15a^3b^3} \div \frac{2xy^2}{5ab^3}$.

$$\frac{4x^2y}{15a^3b^3} \div \frac{2xy^2}{5ab^3} = \frac{4x^2y}{15a^3b^3} \cdot \frac{5ab^3}{2xy^2}$$ Multiply by the reciprocal of the divisor.

$$= \frac{\overset{1}{\cancel{2}} \cdot 2 \cdot \overset{1}{\cancel{x}} \cdot x \cdot \overset{1}{\cancel{y}} \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{a}} \cdot \overset{1}{\cancel{b}} \cdot \overset{1}{\cancel{b}} \cdot \overset{1}{\cancel{b}}}{3 \cdot \underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{a}} \cdot a \cdot a \cdot \underset{1}{\cancel{b}} \cdot \underset{1}{\cancel{b}} \cdot \underset{1}{\cancel{b}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{y}} \cdot y}$$ Factor.

$$= \frac{2 \cdot x}{3 \cdot a \cdot a \cdot y}$$ Simplify.

$$= \frac{2x}{3a^2y}$$ Simplify.

These same steps are followed when the rational expressions contain numerators and denominators that are polynomials.

Example 6 Polynomials in the Numerator and Denominator

Simplify each expression.

a. $\frac{x^2 + 2x - 8}{x^2 + 4x + 3} \cdot \frac{3x + 3}{x - 2}$

$$\frac{x^2 + 2x - 8}{x^2 + 4x + 3} \cdot \frac{3x + 3}{x - 2} = \frac{(x + 4)\overset{1}{\cancel{(x - 2)}}}{(x + 3)\underset{1}{\cancel{(x + 1)}}} \cdot \frac{3\overset{1}{\cancel{(x + 1)}}}{\underset{1}{\cancel{(x - 2)}}} \quad \text{Factor.}$$

$$= \frac{3(x + 4)}{(x + 3)} \quad \text{Simplify.}$$

$$= \frac{3x + 12}{x + 3} \quad \text{Simplify.}$$

b. $\frac{a + 2}{a + 3} \div \frac{a^2 + a - 12}{a^2 - 9}$

$$\frac{a + 2}{a + 3} \div \frac{a^2 + a - 12}{a^2 - 9} = \frac{a + 2}{a + 3} \cdot \frac{a^2 - 9}{a^2 + a - 12} \quad \text{Multiply by the reciprocal of the divisor.}$$

$$= \frac{(a + 2)\overset{1}{\cancel{(a + 3)}}\overset{1}{\cancel{(a - 3)}}}{\underset{1}{\cancel{(a + 3)}}(a + 4)\underset{1}{\cancel{(a - 3)}}} \quad \text{Factor.}$$

$$= \frac{a + 2}{a + 4} \quad \text{Simplify.}$$

Study Tip

Factor First

As in Example 6, sometimes you must factor the numerator and/or the denominator first before you can simplify a quotient of rational expressions.

SIMPLIFY COMPLEX FRACTIONS A **complex fraction** is a rational expression whose numerator and/or denominator contains a rational expression. The expressions below are complex fractions.

$$\frac{\frac{a}{5}}{3b} \qquad \frac{3}{\frac{t}{t + 5}} \qquad \frac{\frac{m^2 - 9}{8}}{\frac{3 - m}{12}} \qquad \frac{\frac{1}{p} + 2}{\frac{3}{p} - 4}$$

Remember that a fraction is nothing more than a way to express a division problem. For example, $\frac{2}{5}$ can be expressed as $2 \div 5$. So to simplify any complex fraction, rewrite it as a division expression and use the rules for division.

Example 7 Simplify a Complex Fraction

Simplify $\frac{\frac{r^2}{r^2 - 25s^2}}{\frac{r}{5s - r}}$.

$$\frac{\frac{r^2}{r^2 - 25s^2}}{\frac{r}{5s - r}} = \frac{r^2}{r^2 - 25s^2} \div \frac{r}{5s - r} \quad \text{Express as a division expression.}$$

$$= \frac{r^2}{r^2 - 25s^2} \cdot \frac{5s - r}{r} \quad \text{Multiply by the reciprocal of the divisor.}$$

$$= \frac{\overset{1}{\cancel{r}} \cdot r(-1)\overset{1}{\cancel{(r - 5s)}}}{(r + 5s)\underset{1}{\cancel{(r - 5s)}}\underset{1}{\cancel{r}}} \quad \text{Factor.}$$

$$= \frac{-r}{r + 5s} \text{ or } -\frac{r}{r + 5s} \quad \text{Simplify.}$$

Check for Understanding

Concept Check

1. **OPEN ENDED** Write two rational expressions that are equivalent.
2. **Explain** how multiplication and division of rational expressions are similar to multiplication and division of rational numbers.
3. **Determine** whether $\frac{2d+5}{3d+5} = \frac{2}{3}$ is *sometimes*, *always*, or *never* true. Explain.

Guided Practice

Simplify each expression.

4. $\frac{45mn^3}{20n^7}$
5. $\frac{a+b}{a^2-b^2}$
6. $\frac{6y^3-9y^2}{2y^2+5y-12}$
7. $\frac{2a^2}{5b^2c} \cdot \frac{3bc^2}{8a^2}$
8. $\frac{35}{16x^2} \div \frac{21}{4x}$
9. $\frac{3t+6}{7t-7} \cdot \frac{14t-14}{5t+10}$
10. $\frac{12p^2+6p-6}{4(p+1)^2} \div \frac{6p-3}{2p+10}$
11. $\dfrac{\frac{c^3d^3}{a}}{\frac{xc^2d}{ax^2}}$
12. $\dfrac{\frac{2y}{y^2-4}}{\frac{3}{y^2-4y+4}}$

Standardized Test Practice

13. Identify all of the values of y for which the expression $\frac{y-4}{y^2-4y-12}$ is undefined.

(A) −2, 4, 6 (B) −6, −4, 2 (C) −2, 0, 6 (D) −2, 6

Practice and Apply

Homework Help

For Exercises	See Examples
14–21	1, 3
22–35	4–6
36–41	7
42, 43, 50	2

Extra Practice

See page 847.

Simplify each expression.

14. $\frac{30bc}{12b^2}$
15. $\frac{-3mn^4}{21m^2n^2}$
16. $\frac{(-3x^2y)^3}{9x^2y^2}$
17. $\frac{(-2rs^2)^2}{12r^2s^3}$
18. $\frac{5t-5}{t^2-1}$
19. $\frac{c+5}{2c+10}$
20. $\frac{y^2+4y+4}{3y^2+5y-2}$
21. $\frac{a^2+2a+1}{2a^2+3a+1}$
22. $\frac{3xyz}{4xz} \cdot \frac{6x^2}{3y^2}$
23. $\frac{-4ab}{21c} \cdot \frac{14c^2}{18a^2}$
24. $\frac{3}{5d} \div \left(\frac{-9}{15df}\right)$
25. $\frac{p^3}{2q} \div \frac{-p}{4q}$
26. $\frac{2x^3y}{z^5} \div \left(\frac{4xy}{z^3}\right)^2$
27. $\frac{xy}{a^3} \div \left(\frac{xy}{ab}\right)^3$
28. $\frac{3t^2}{t+2} \cdot \frac{t+2}{t^2}$
29. $\frac{4w+4}{3} \cdot \frac{1}{w+1}$
30. $\frac{4t^2-4}{9(t+1)^2} \cdot \frac{3t+3}{2t-2}$
31. $\frac{3p-21}{p^2-49} \cdot \frac{p^2+7p}{3p}$
32. $\frac{5x^2+10x-75}{4x^2-24x-28} \cdot \frac{2x^2-10x-28}{x^2+7x+10}$
33. $\frac{w^2-11w+24}{w^2-18w+80} \cdot \frac{w^2-15w+50}{w^2-9w+20}$
34. $\frac{r^2+2r-8}{r^2+4r+3} \div \frac{r-2}{3r+3}$
35. $\frac{a^2+2a-15}{a-3} \div \frac{a^2-4}{2}$

36. $\dfrac{\frac{m^3}{3n}}{-\frac{m^4}{9n^2}}$

37. $\dfrac{\frac{p^3}{2q}}{\frac{-p^2}{4q}}$

38. $\dfrac{\frac{m+n}{5}}{\frac{m^2+n^2}{5}}$

39. $\dfrac{\frac{x+y}{2x-y}}{\frac{x+y}{2x+y}}$

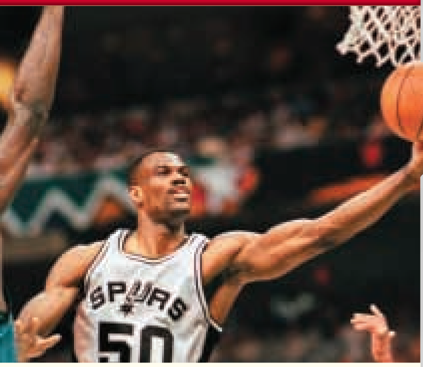
40. $\dfrac{\frac{6y^2-6}{8y^2+8y}}{\frac{3y-3}{4y^2+4y}}$

41. $\dfrac{\frac{5x^2-5x-30}{45-15x}}{\frac{6+x-x^2}{4x-12}}$

42. Under what conditions is $\dfrac{2d(d+1)}{(d+1)(d^2-4)}$ undefined?

43. Under what conditions is $\dfrac{a^2+ab+b^2}{a^2-b^2}$ undefined?

More About. . .



Basketball

After graduating from the U.S. Naval Academy, David Robinson became the NBA Rookie of the Year in 1990. He has played basketball in 3 different Olympic Games.

Source: NBA

BASKETBALL **For Exercises 44 and 45, use the following information.**
At the end of the 2000–2001 season, David Robinson had made 6827 field goals out of 13,129 attempts during his NBA career.

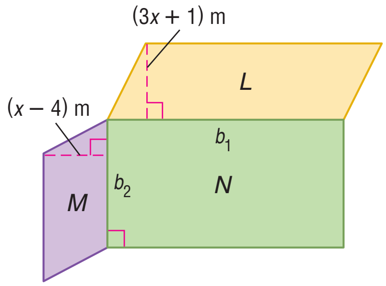
44. Write a fraction to represent the ratio of the number of career field goals made to career field goals attempted by David Robinson at the end of the 2000–2001 season.

45. Suppose David Robinson attempted a field goals and made m field goals during the 2001–2002 season. Write a rational expression to represent the number of career field goals made to the number of career field goals attempted at the end of the 2001–2002 season.

Online Research **Data Update** What are the current scoring statistics of your favorite NBA player? Visit www.algebra2.com/data_update to learn more.

46. GEOMETRY A parallelogram with an area of $6x^2 - 7x - 5$ square units has a base of $3x - 5$ units. Determine the height of the parallelogram.

47. GEOMETRY Parallelogram L has an area of $3x^2 + 10x + 3$ square meters and a height of $3x + 1$ meters. Parallelogram M has an area of $2x^2 - 13x + 20$ square meters and a height of $x - 4$ meters. Find the area of rectangle N.



48. CRITICAL THINKING Simplify $\dfrac{(a^2-5a+6)^{-1}}{(a-2)^{-2}} \div \dfrac{(a-3)^{-1}}{(a-2)^{-2}}$.

49. *WRITING IN MATH* Answer the question that was posed at the beginning of the lesson.

How are rational expressions used in mixtures?

Include the following in your answer:

- an explanation of how to determine whether the rational expression representing the nut mixture is in simplest form, and
- an example of a mixture problem that could be represented by $\dfrac{8+x}{13+x+y}$.

Standardized Test Practice

50. For what value(s) of x is the expression $\frac{4x}{x^2 - x}$ undefined?

Ⓐ $-1, 1$ Ⓑ $-1, 0, 1$ Ⓒ $0, 1$ Ⓓ 0 Ⓔ $1, 2$

51. Compare the quantity in Column A and the quantity in Column B. Then determine whether:

Ⓐ **the quantity in Column A is greater,**

Ⓑ **the quantity in Column B is greater,**

Ⓒ **the two quantities are equal, or**

Ⓓ **the relationship cannot be determined from the information given.**

Column A	Column B
$\frac{a^2 + 3a - 10}{a - 2}$	$\frac{a^2 + a - 6}{a + 3}$

Maintain Your Skills

Mixed Review

Find the exact solution(s) of each system of equations. *(Lesson 8–7)*

52. $x^2 + 2y^2 = 33$
$x^2 + y^2 - 19 = 2x$

53. $x^2 + 2y^2 = 33$
$x^2 - y^2 = 9$

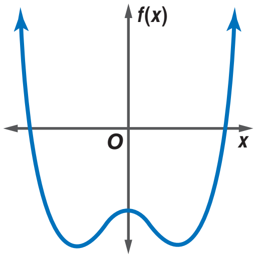
Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation. *(Lesson 8–6)*

54. $y^2 - 3x + 6y + 12 = 0$

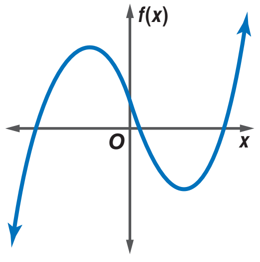
55. $x^2 - 14x + 4 = 9y^2 - 36y$

Determine whether each graph represents an odd-degree function or an even-degree function. Then state how many real zeros each function has. *(Lesson 7–1)*

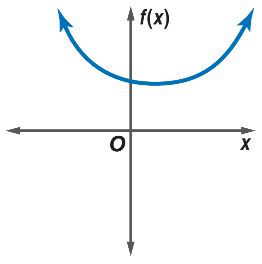
56.



57.



58.



Solve each equation by factoring. *(Lesson 6–3)*

59. $r^2 - 3r = 4$

60. $18u^2 - 3u = 1$

61. $d^2 - 5d = 0$

62. ASTRONOMY Earth is an average 1.496×10^8 kilometers from the Sun. If light travels 3×10^5 kilometers per second, how long does it take sunlight to reach Earth? *(Lesson 5–1)*

Solve each equation. *(Lesson 1–4)*

63. $|2x + 7| + 5 = 0$

64. $5|3x - 4| = x + 1$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation.** *(To review **solving equations,** see Lesson 1–3.)*

65. $\frac{2}{3} + x = -\frac{4}{9}$

66. $x + \frac{5}{8} = -\frac{5}{6}$

67. $x - \frac{3}{5} = \frac{2}{3}$

68. $x + \frac{3}{16} = -\frac{1}{2}$

69. $x - \frac{1}{6} = -\frac{7}{9}$

70. $x - \frac{3}{8} = -\frac{5}{24}$

9-2 Adding and Subtracting Rational Expressions

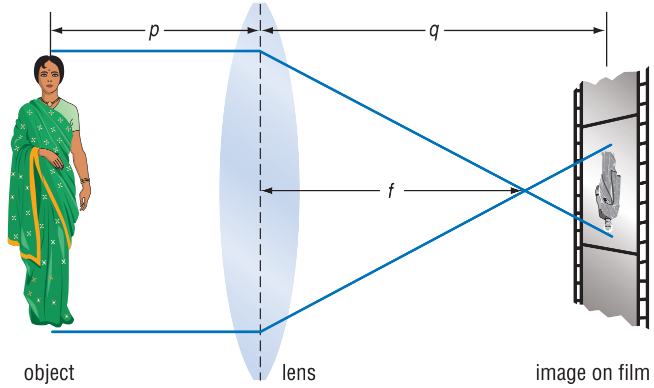
What You'll Learn

- Determine the LCM of polynomials.
- Add and subtract rational expressions.

How is subtraction of rational expressions used in photography?

To take sharp, clear pictures, a photographer must focus the camera precisely. The distance from the object to the lens p and the distance from the lens to the film q must be accurately calculated to ensure a sharp image. The focal length of the lens is f.

The formula $\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$ can be used to determine how far the film should be placed from the lens to create a perfect photograph.



LCM OF POLYNOMIALS To find $\frac{5}{6} - \frac{1}{4}$ or $\frac{1}{f} - \frac{1}{p}$, you must first find the least common denominator (LCD). The LCD is the least common multiple (LCM) of the denominators.

To find the LCM of two or more numbers or polynomials, factor each number or polynomial. The LCM contains *each* factor the greatest number of times it appears as a factor.

LCM of 6 and 4	LCM of $a^2 - 6a + 9$ and $a^2 + a - 12$
$6 = 2 \cdot 3$	$a^2 - 6a + 9 = (a - 3)^2$
$4 = 2^2$	$a^2 + a - 12 = (a - 3)(a + 4)$
$\text{LCM} = 2^2 \cdot 3$ or 12	$\text{LCM} = (a - 3)^2(a + 4)$

Example 1 LCM of Monomials

Find the LCM of $18r^2s^5$, $24r^3st^2$, and $15s^3t$.

$18r^2s^5 = 2 \cdot 3^2 \cdot r^2 \cdot s^5$ — Factor the first monomial.

$24r^3st^2 = 2^3 \cdot 3 \cdot r^3 \cdot s \cdot t^2$ — Factor the second monomial.

$15s^3t = 3 \cdot 5 \cdot s^3 \cdot t$ — Factor the third monomial.

$\text{LCM} = 2^3 \cdot 3^2 \cdot 5 \cdot r^3 \cdot s^5 \cdot t^2$ — Use each factor the greatest number of times it appears as a factor and simplify.

$= 360r^3s^5t^2$

Example 2 LCM of Polynomials

Find the LCM of $p^3 + 5p^2 + 6p$ and $p^2 + 6p + 9$.

$p^3 + 5p^2 + 6p = p(p + 2)(p + 3)$ — Factor the first polynomial.

$p^2 + 6p + 9 = (p + 3)^2$ — Factor the second polynomial.

$\text{LCM} = p(p + 2)(p + 3)^2$ — Use each factor the greatest number of times it appears as a factor.

ADD AND SUBTRACT RATIONAL EXPRESSIONS As with fractions, to add or subtract rational expressions, you must have common denominators.

Specific Case		General Case
$\frac{2}{3} + \frac{3}{5} = \frac{2 \cdot 5}{3 \cdot 5} + \frac{3 \cdot 3}{5 \cdot 3}$	Find equivalent fractions that have a common denominator.	$\frac{a}{c} + \frac{b}{d} = \frac{a \cdot d}{c \cdot d} + \frac{b \cdot c}{d \cdot c}$
$= \frac{10}{15} + \frac{9}{15}$	Simplify each numerator and denominator.	$= \frac{ad}{cd} + \frac{bc}{cd}$
$= \frac{19}{15}$	Add the numerators.	$= \frac{ad + bc}{cd}$

Example 3 Monomial Denominators

Simplify $\frac{7x}{15y^2} + \frac{y}{18xy}$.

$\frac{7x}{15y^2} + \frac{y}{18xy} = \frac{7x \cdot 6x}{15y^2 \cdot 6x} + \frac{y \cdot 5y}{18xy \cdot 5y}$ — The LCD is $90xy^2$. Find equivalent fractions that have this denominator.

$= \frac{42x^2}{90xy^2} + \frac{5y^2}{90xy^2}$ — Simplify each numerator and denominator.

$= \frac{42x^2 + 5y^2}{90xy^2}$ — Add the numerators.

Example 4 Polynomial Denominators

Simplify $\frac{w + 12}{4w - 16} - \frac{w + 4}{2w - 8}$.

$\frac{w + 12}{4w - 16} - \frac{w + 4}{2w - 8} = \frac{w + 12}{4(w - 4)} - \frac{w + 4}{2(w - 4)}$ — Factor the denominators.

$= \frac{w + 12}{4(w - 4)} - \frac{(w + 4)(2)}{2(w - 4)(2)}$ — The LCD is $4(w-4)$.

$= \frac{(w + 12) - (2)(w + 4)}{4(w - 4)}$ — Subtract the numerators.

$= \frac{w + 12 - 2w - 8}{4(w - 4)}$ — Distributive Property

$= \frac{-w + 4}{4(w - 4)}$ — Combine like terms.

$= \frac{-1\overset{1}{\cancel{(w - 4)}}}{4\underset{1}{\cancel{(w - 4)}}}$ or $-\frac{1}{4}$ — Simplify.

Study Tip

Common Factors

Sometimes when you simplify the numerator, the polynomial contains a factor common to the denominator. Thus, the rational expression can be further simplified.

Sometimes simplifying complex fractions involves adding or subtracting rational expressions. One way to simplify a complex fraction is to simplify the numerator and the denominator separately, and then simplify the resulting expressions.

Example 5 Simplify Complex Fractions

Simplify $\dfrac{\frac{1}{x} - \frac{1}{y}}{1 + \frac{1}{x}}$.

$$\frac{\frac{1}{x} - \frac{1}{y}}{1 + \frac{1}{x}} = \frac{\frac{y}{xy} - \frac{x}{xy}}{\frac{x}{x} + \frac{1}{x}}$$ The LCD of the numerator is xy. The LCD of the denominator is x.

$$= \frac{\frac{y - x}{xy}}{\frac{x + 1}{x}}$$ Simplify the numerator and denominator.

$$= \frac{y - x}{xy} \div \frac{x + 1}{x}$$ Write as a division expression.

$$= \frac{y - x}{\cancel{x}^{1}y} \cdot \frac{\cancel{x}^{1}}{x + 1}$$ Multiply by the reciprocal of the divisor.

$$= \frac{y - x}{y(x + 1)} \text{ or } \frac{y - x}{xy + y}$$ Simplify.

Example 6 Use a Complex Fraction to Solve a Problem

COORDINATE GEOMETRY Find the slope of the line that passes through $A\left(\frac{2}{p}, \frac{1}{2}\right)$ and $B\left(\frac{1}{3}, \frac{3}{p}\right)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Definition of slope

$$= \frac{\frac{3}{p} - \frac{1}{2}}{\frac{1}{3} - \frac{2}{p}}$$ $y_2 = \frac{3}{p}$, $y_1 = \frac{1}{2}$, $x_2 = \frac{1}{3}$, and $x_1 = \frac{2}{p}$

$$= \frac{\frac{6 - p}{2p}}{\frac{p - 6}{3p}}$$ The LCD of the numerator is $2p$. The LCD of the denominator is $3p$.

$$= \frac{6 - p}{2p} \div \frac{p - 6}{3p}$$ Write as a division expression.

$$= \frac{\cancel{6 - p}^{-1}}{2\cancel{p}_{1}} \cdot \frac{3\cancel{p}^{1}}{\cancel{p - 6}_{1}} \text{ or } -\frac{3}{2}$$ The slope is $-\frac{3}{2}$.

Study Tip

Check Your Solution

You can check your answer be letting p equal any nonzero number, say 1. Use the definition of slope to find the slope of the line through the points.

Check for Understanding

Concept Check

1. **FIND THE ERROR** Catalina and Yong-Chan are simplifying $\frac{x}{a} - \frac{x}{b}$.

Catalina

$$\frac{x}{a} - \frac{x}{b} = \frac{bx}{ab} - \frac{ax}{ab}$$
$$= \frac{bx - ax}{ab}$$

Yong-Chan

$$\frac{x}{a} - \frac{x}{b} = \frac{x}{a - b}$$

Who is correct? Explain your reasoning.

2. **OPEN ENDED** Write two polynomials that have a LCM of $d^3 - d$.

3. Consider $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ if a, b, and c are real numbers. Determine whether each statement is *sometimes*, *always*, or *never* true. Explain your answer.
   a. abc is a common denominator.
   b. abc is the LCD.
   c. ab is the LCD.
   d. b is the LCD.
   e. The sum is $\frac{bc + ac + ab}{abc}$.

Guided Practice

Find the LCM of each set of polynomials.

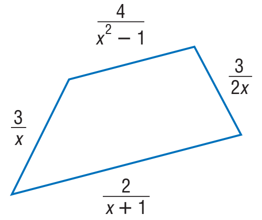
4. $12y^2, 6x^2$
5. $16ab^3, 5b^2a^2, 20ac$
6. $x^2 - 2x, x^2 - 4$

Simplify each expression.

7. $\frac{2}{x^2y} - \frac{x}{y}$
8. $\frac{7a}{15b^2} + \frac{b}{18ab}$
9. $\frac{5}{3m} - \frac{2}{7m} - \frac{1}{2m}$
10. $\frac{6}{d^2 + 4d + 4} + \frac{5}{d + 2}$
11. $\frac{a}{a^2 - a - 20} + \frac{2}{a + 4}$
12. $\dfrac{x + \frac{x}{3}}{x - \frac{x}{6}}$

Application

13. **GEOMETRY** Find the perimeter of the quadrilateral. Express in simplest form.



Practice and Apply

Homework Help

For Exercises	See Examples
14–21	1, 2
22–39	3, 4
40–43	5
44–49	6

Extra Practice

See page 847.

Find the LCM of each set of polynomials.

14. $10s^2, 35s^2t^2$
15. $36x^2y, 20xyz$
16. $14a^3, 15bc^3, 12b^3$
17. $9p^2q^3, 6pq^4, 4p^3$
18. $4w - 12, 2w - 6$
19. $x^2 - y^2, x^3 + x^2y$
20. $2t^2 + t - 3, 2t^2 + 5t + 3$
21. $n^2 - 7n + 12, n^2 - 2n - 8$

Simplify each expression.

22. $\frac{6}{ab} + \frac{8}{a}$
23. $\frac{5}{6v} + \frac{7}{4v}$
24. $\frac{5}{r} + 7$
25. $\frac{2x}{3y} + 5$
26. $\frac{3x}{4y^2} - \frac{y}{6x}$
27. $\frac{5}{a^2b} - \frac{7a}{5b^2}$
28. $\frac{3}{4q} - \frac{2}{5q} - \frac{1}{2q}$
29. $\frac{11}{9} - \frac{7}{2w} - \frac{6}{5w}$
30. $\frac{7}{y - 8} - \frac{6}{8 - y}$
31. $\frac{a}{a - 4} - \frac{3}{4 - a}$
32. $\frac{m}{m^2 - 4} + \frac{2}{3m + 6}$
33. $\frac{y}{y + 3} - \frac{6y}{y^2 - 9}$

CONTENTS

34. $\dfrac{5}{x^2 - 3x - 28} + \dfrac{7}{2x - 14}$

35. $\dfrac{d - 4}{d^2 + 2d - 8} - \dfrac{d + 2}{d^2 - 16}$

36. $\dfrac{1}{h^2 - 9h + 20} - \dfrac{5}{h^2 - 10h + 25}$

37. $\dfrac{x}{x^2 + 5x + 6} - \dfrac{2}{x^2 + 4x + 4}$

38. $\dfrac{m^2 + n^2}{m^2 - n^2} + \dfrac{m}{n - m} + \dfrac{n}{m + n}$

39. $\dfrac{y + 1}{y - 1} + \dfrac{y + 2}{y - 2} + \dfrac{y}{y^2 - 3y + 2}$

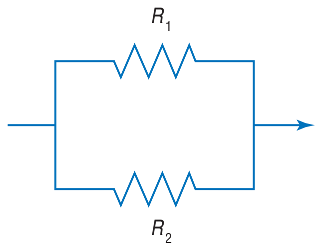
40. $\dfrac{\dfrac{1}{b + 2} + \dfrac{1}{b - 5}}{\dfrac{2b^2 - b - 3}{b^2 - 3b - 10}}$

41. $\dfrac{(x + y)\left(\dfrac{1}{x} - \dfrac{1}{y}\right)}{(x - y)\left(\dfrac{1}{x} + \dfrac{1}{y}\right)}$

42. Write $\left(\dfrac{2s}{2s + 1} - 1\right) \div \left(1 + \dfrac{2s}{1 - 2s}\right)$ in simplest form.

43. What is the simplest form of $\left(3 + \dfrac{5}{a + 2}\right) \div \left(3 - \dfrac{10}{a + 7}\right)$?

ELECTRICITY **For Exercises 44 and 45, use the the following information.**
In an electrical circuit, if two resistors with resistance R_1 and R_2 are connected in parallel as shown, the relationship between these resistances and the resulting combination resistance R is $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$.



44. If R_1 is x ohms and R_2 is 4 ohms less than twice x ohms, write an expression for $\dfrac{1}{R}$.

45. Find the effective resistance of a 30-ohm resistor and a 20-ohm resistor that are connected in parallel.

More About. . .

Bicycling
The Tour de France is the most popular bicycle road race. It lasts 24 days and covers 2500 miles.
Source: *World Book Encyclopedia*

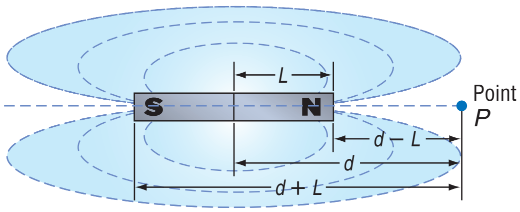
BICYCLING **For Exercises 46–48, use the following information.**
Jalisa is competing in a 48-mile bicycle race. She travels half the distance at one rate. The rest of the distance, she travels 4 miles per hour slower.

46. If x represents the faster pace in miles per hour, write an expression that represents the time spent at that pace.

47. Write an expression for the amount of time spent at the slower pace.

48. Write an expression for the amount of time Jalisa needed to complete the race.

49. **MAGNETS** For a bar magnet, the magnetic field strength H at a point P along the axis of the magnet is $H = \dfrac{m}{2L(d - L)^2} - \dfrac{m}{2L(d + L)^2}$. Write a simpler expression for H.



50. **CRITICAL THINKING** Find two rational expressions whose sum is $\dfrac{2x - 1}{(x + 1)(x - 2)}$.

51. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is subtraction of rational expressions used in photography?
Include the following in your answer:
- an explanation of how to subtract rational expressions, and
- an equation that could be used to find the distance between the lens and the film if the focal length of the lens is 10 centimeters and the distance between the lens and the object is 60 centimeters.

Standardized Test Practice

52. For all $t \neq 5$, $\frac{t^2 - 25}{3t - 15} =$

Ⓐ $\frac{t-5}{3}$. Ⓑ $\frac{t+5}{3}$. Ⓒ $t - 5$. Ⓓ $t + 5$. Ⓔ $\frac{t-5}{t-3}$.

53. What is the sum of $\frac{x-y}{5}$ and $\frac{x+y}{4}$?

Ⓐ $\frac{9x+9y}{20}$ Ⓑ $\frac{x+9y}{20}$ Ⓒ $\frac{9x+y}{20}$ Ⓓ $\frac{9x-y}{20}$ Ⓔ $\frac{x-9y}{20}$

Maintain Your Skills

Mixed Review

Simplify each expression. *(Lesson 9-1)*

54. $\frac{9x^2y^3}{(5xyz)^2} \div \frac{(3xy)^3}{20x^2y}$

55. $\frac{5a^2 - 20}{2a + 2} \cdot \frac{4a}{10a - 20}$

Solve each system of inequalities by graphing. *(Lesson 8-7)*

56. $9x^2 + y^2 < 81$
$x^2 + y^2 \geq 16$

57. $(y - 3)^2 \geq x + 2$
$x^2 \leq y + 4$

58. **GARDENS** Helene Jonson has a rectangular garden 25 feet by 50 feet. She wants to increase the garden on all sides by an equal amount. If the area of the garden is to be increased by 400 square feet, by how much should each dimension be increased? *(Lesson 6-4)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Draw the asymptotes and graph each hyperbola.**
*(To review **graphing hyperbolas,** see Lesson 8-5.)*

59. $\frac{x^2}{16} - \frac{y^2}{20} = 1$

60. $\frac{y^2}{49} - \frac{x^2}{25} = 1$

61. $\frac{(x+2)^2}{16} - \frac{(y-5)^2}{25} = 1$

Practice Quiz 1

Lessons 9-1 and 9-2

Simplify each expression. *(Lesson 9-1)*

1. $\frac{t^2 - t - 6}{t^2 - 6t + 9}$

2. $\frac{3ab^3}{8a^2b} \cdot \frac{4ac}{9b^4}$

3. $-\frac{4}{8x} \div \frac{16}{xy^2}$

4. $\frac{48}{6a + 42} \cdot \frac{7a + 49}{16}$

5. $\frac{w^2 + 5w + 4}{6} \div \frac{w + 1}{18w + 24}$

6. $\frac{\frac{x^2 + x}{x + 1}}{\frac{x}{x - 1}}$

Simplify each expression. *(Lesson 9-2)*

7. $\frac{4a + 2}{a + b} + \frac{1}{-b - a}$

8. $\frac{2x}{5ab^3} + \frac{4y}{3a^2b^2}$

9. $\frac{5}{n + 6} - \frac{4}{n - 1}$

10. $\frac{x - 5}{2x - 6} - \frac{x - 7}{4x - 12}$

9-3 Graphing Rational Functions

What You'll Learn

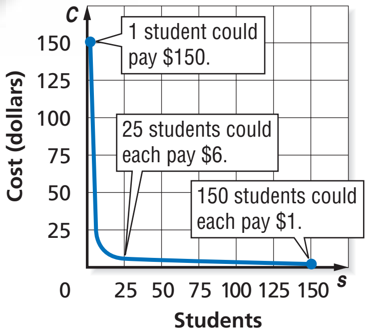
- Determine the vertical asymptotes and the point discontinuity for the graphs of rational functions.
- Graph rational functions.

Vocabulary

- rational function
- continuity
- asymptote
- point discontinuity

How can rational functions be used when buying a group gift?

A group of students want to get their favorite teacher, Mr. Salgado, a retirement gift. They plan to get him a gift certificate for a weekend package at a lodge in a state park. The certificate costs $150. If c represents the cost for each student and s represents the number of students, then $c = \frac{150}{s}$.



VERTICAL ASYMPTOTES AND POINT DISCONTINUITY The function $c = \frac{150}{s}$ is an example of a rational function. A **rational function** is an equation of the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial functions and $q(x) \neq 0$. Here are other examples of rational functions.

$$f(x) = \frac{x}{x + 3} \qquad g(x) = \frac{5}{x - 6} \qquad h(x) = \frac{x + 4}{(x - 1)(x + 4)}$$

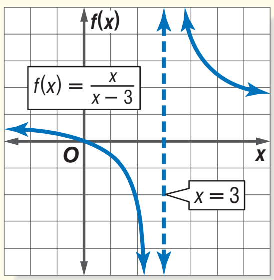
No denominator in a rational function can be zero because division by zero is not defined. In the examples above, the functions are not defined at $x = -3$, $x = 6$, and $x = 1$ and $x = -4$, respectively.

Study Tip

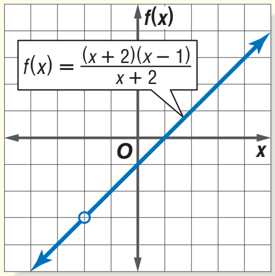
Look Back
To review **asymptotes**, see Lesson 8-5.

The graphs of rational functions may have breaks in **continuity**. This means that, unlike polynomial functions, which can be traced with a pencil never leaving the paper, not all rational functions are traceable. Breaks in continuity can appear as a vertical **asymptote** or as a **point discontinuity**. Recall that an asymptote is a line that the graph of the function approaches, but never crosses. Point discontinuity is like a hole in a graph.

Key Concept — *Vertical Asymptotes*

Property	Words	Example	Model
Vertical Asymptote	If the rational expression of a function is written in simplest form and the function is undefined for $x = a$, then $x = a$ is a vertical asymptote.	For $f(x) = \frac{x}{x - 3}$, $x = 3$ is a vertical asymptote.	

Key Concept — Point Discontinuity

Property	Words	Example	Model
Point Discontinuity	If the original function is undefined for $x = a$ but the rational expression of the function in simplest form is defined for $x = a$, then there is a hole in the graph at $x = a$.	$f(x) = \frac{(x+2)(x-1)}{x+2}$ can be simplified to $f(x) = x - 1$. So, $x = -2$ represents a hole in the graph.	

Example 1 Vertical Asymptotes and Point Discontinuity

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of $f(x) = \frac{x^2 - 1}{x^2 - 6x + 5}$.

First factor the numerator and denominator of the rational expression.

$$\frac{x^2 - 1}{x^2 - 6x + 5} = \frac{(x-1)(x+1)}{(x-1)(x-5)}$$

The function is undefined for $x = 1$ and $x = 5$. Since $\frac{\overset{1}{\cancel{(x-1)}}(x+1)}{\underset{1}{\cancel{(x-1)}}(x-5)} = \frac{x+1}{x-5}$,

$x = 5$ is a vertical asymptote, and $x = 1$ represents a hole in the graph.

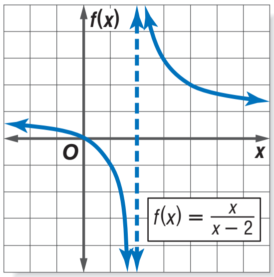
GRAPH RATIONAL FUNCTIONS You can use what you know about vertical asymptotes and point discontinuity to graph rational functions.

Example 2 Graph with a Vertical Asymptote

Graph $f(x) = \frac{x}{x-2}$.

The function is undefined for $x = 2$. Since $\frac{x}{x-2}$ is in simplest form, $x = 2$ is a vertical asymptote. Draw the vertical asymptote. Make a table of values. Plot the points and draw the graph.

x	$f(x)$
−50	0.96154
−30	0.9375
−20	0.90909
−10	0.83333
−2	0.5
−1	0.33333
0	0
1	−1
3	3
4	2
5	1.6667
10	1.25
20	1.1111
30	1.0714
50	1.0417



As $|x|$ increases, it appears that the y values of the function get closer and closer to 1. The line with the equation $f(x) = 1$ is a horizontal asymptote of the function.

Study Tip

Graphing Rational Functions

Finding the x- and y-intercepts is often useful when graphing rational functions.

CONTENTS

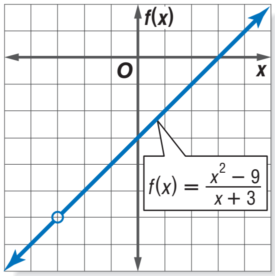
As you have learned, graphs of rational functions may have point discontinuity rather than vertical asymptotes. The graphs of these functions appear to have holes. These holes are usually shown as circles on graphs.

Example 3 Graph with Point Discontinuity

Graph $f(x) = \frac{x^2 - 9}{x + 3}$.

Notice that $\frac{x^2 - 9}{x + 3} = \frac{(x + 3)(x - 3)}{x + 3}$ or $x - 3$.

Therefore, the graph of $f(x) = \frac{x^2 - 9}{x + 3}$ is the graph of $f(x) = x - 3$ with a hole at $x = -3$.

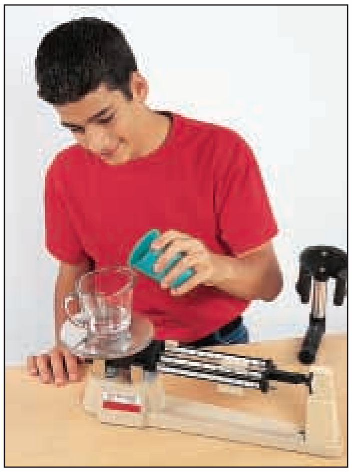


Many real-life situations can be described by using rational functions.

Algebra Activity

Rational Functions

The density of a material can be expressed as $D = \frac{m}{V}$, where m is the mass of the material in grams and V is the volume in cubic centimeters. By finding the volume and density of 200 grams of each liquid, you can draw a graph of the function $D = \frac{200}{V}$.



Collect the Data

- Use a balance and metric measuring cups to find the volumes of 200 grams of different liquids such as water, cooking oil, isopropyl alcohol, sugar water, and salt water.
- Use $D = \frac{m}{V}$ to find the density of each liquid.

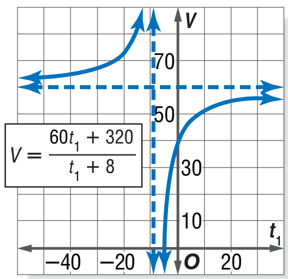
Analyze the Data

1. Graph the data by plotting the points (volume, density) on a graph. Connect the points.
2. From the graph, find the asymptotes.

In the real world, sometimes values on the graph of a rational function are not meaningful.

Example 4 Use Graphs of Rational Functions

TRANSPORTATION **A train travels at one velocity V_1 for a given amount of time t_1 and then another velocity V_2 for a different amount of time t_2. The average velocity is given by $V = \frac{V_1 t_1 + V_2 t_2}{t_1 + t_2}$.**



a. Let t_1 be the independent variable and let V be the dependent variable. Draw the graph if $V_1 = 60$ miles per hour, $V_2 = 40$ miles per hour, and $t_2 = 8$ hours.

The function is $V = \frac{60t_1 + 40(8)}{t_1 + 8}$ or $V = \frac{60t_1 + 320}{t_1 + 8}$. The vertical asymptote is $t_1 = -8$. Graph the vertical asymptote and the function. Notice that the horizontal asymptote is $V = 60$.

b. What is the V-intercept of the graph?

The V-intercept is 40.

c. What values of t_1 and V are meaningful in the context of the problem?

In the problem context, time and velocity are positive values. Therefore, only values of t_1 greater than 0 and values of V between 40 and 60 are meaningful.

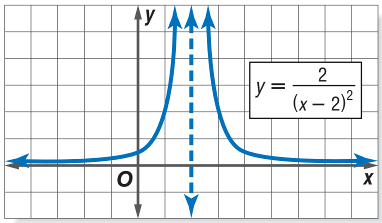
Check for Understanding

Concept Check

1. **OPEN ENDED** Write a function whose graph has two vertical asymptotes located at $x = -5$ and $x = 2$.

2. **Compare and contrast** the graphs of $f(x) = \frac{(x-1)(x+5)}{x-1}$ and $g(x) = x + 5$.

3. **Describe** the graph at the right. Include the equations of any asymptotes, the x values of any holes, and the x- and y-intercepts.



Guided Practice

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

4. $f(x) = \frac{3}{x^2 - 4x + 4}$
5. $f(x) = \frac{x-1}{x^2 + 4x - 5}$

Graph each rational function.

6. $f(x) = \frac{x}{x+1}$
7. $f(x) = \frac{6}{(x-2)(x+3)}$
8. $f(x) = \frac{x^2 - 25}{x - 5}$
9. $f(x) = \frac{x-5}{x+1}$
10. $f(x) = \frac{4}{(x-1)^2}$
11. $f(x) = \frac{x+2}{x^2 - x - 6}$

Application

MEDICINE **For Exercises 12–15, use the following information.**
For certain medicines, health care professionals may use Young's Rule, $C = \frac{y}{y+12} \cdot D$, to estimate the proper dosage for a child when the adult dosage is known. In this equation, C represents the child's dose, D represents the adult dose, and y represents the child's age in years.

12. Use Young's Rule to estimate the dosage of amoxicillin for an eight-year-old child if the adult dosage is 250 milligrams.

13. Graph $C = \frac{y}{y+12}$.

14. Give the equations of any asymptotes and y- and C-intercepts of the graph.

15. What values of y and C are meaningful in the context of the problem?

Practice and Apply

Homework Help

For Exercises	See Examples
16–21	1
22–39	2, 3
40–50	4

Extra Practice

See page 849.

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

16. $f(x) = \frac{2}{x^2 - 5x + 6}$

17. $f(x) = \frac{4}{x^2 + 2x - 8}$

18. $f(x) = \frac{x + 3}{x^2 + 7x + 12}$

19. $f(x) = \frac{x - 5}{x^2 - 4x - 5}$

20. $f(x) = \frac{x^2 - 8x + 16}{x - 4}$

21. $f(x) = \frac{x^2 - 3x + 2}{x - 1}$

Graph each rational function.

22. $f(x) = \frac{1}{x}$

23. $f(x) = \frac{3}{x}$

24. $f(x) = \frac{1}{x + 2}$

25. $f(x) = \frac{-5}{x + 1}$

26. $f(x) = \frac{x}{x - 3}$

27. $f(x) = \frac{5x}{x + 1}$

28. $f(x) = \frac{-3}{(x - 2)^2}$

29. $f(x) = \frac{1}{(x + 3)^2}$

30. $f(x) = \frac{x + 4}{x - 1}$

31. $f(x) = \frac{x - 1}{x - 3}$

32. $f(x) = \frac{x^2 - 36}{x + 6}$

33. $f(x) = \frac{x^2 - 1}{x - 1}$

34. $f(x) = \frac{3}{(x - 1)(x + 5)}$

35. $f(x) = \frac{-1}{(x + 2)(x - 3)}$

36. $f(x) = \frac{x}{x^2 - 1}$

37. $f(x) = \frac{x - 1}{x^2 - 4}$

38. $f(x) = \frac{6}{(x - 6)^2}$

39. $f(x) = \frac{1}{(x + 2)^2}$

HISTORY **For Exercises 40–42, use the following information.**
In Maria Gaetana Agnesi's book *Analytical Institutions*, Agnesi discussed the characteristics of the equation $x^2y = a^2(a - y)$, whose graph is called the "curve of Agnesi." This equation can be expressed as $y = \frac{a^3}{x^2 + a^2}$.

40. Graph $f(x) = \frac{a^3}{x^2 + a^2}$ if $a = 4$.

41. Describe the graph.

42. **Make a conjecture** about the shape of the graph of $f(x) = \frac{a^3}{x^2 + a^2}$ if $a = -4$. Explain your reasoning.

More About. . .

History
Mathematician Maria Gaetana Agnesi was one of the greatest scholars of all time. Born in Milan, Italy, in 1718, she mastered Greek, Hebrew, and several modern languages by the age of 11.

Source: *A History of Mathematics*

AUTO SAFETY **For Exercises 43–45, use the following information.**
When a car has a front-end collision, the objects in the car (including passengers) keep moving forward until the impact occurs. After impact, objects are repelled. Seat belts and airbags limit how far you are jolted forward. The formula for the velocity at which you are thrown backward is $V_f = \frac{(m_1 - m_2)v_i}{m_1 + m_2}$, where m_1 and m_2 are masses of the two objects meeting and v_i is the initial velocity.

43. Let m_1 be the independent variable, and let V_f be the dependent variable. Graph the function if $m_2 = 7$ kilograms and $v_i = 5$ meters per second.

44. Give the equation of the vertical asymptote and the m_1- and V_f-intercepts of the graph.

45. Find the value of V_f when the value of m_1 is 5 kilograms.

46. **CRITICAL THINKING** Write three rational functions that have a vertical asymptote at $x = 3$ and a hole at $x = -2$.

www.algebra2.com/self_check_quiz

BASKETBALL **For Exercises 47–50, use the following information.**
Zonta plays basketball for Centerville High School. So far this season, she has made 6 out of 10 free throws. She is determined to improve her free-throw percentage. If she can make x consecutive free throws, her free-throw percentage can be determined using $P(x) = \frac{6 + x}{10 + x}$.

47. Graph the function.

48. What part of the graph is meaningful in the context of the problem?

49. Describe the meaning of the y-intercept.

50. What is the equation of the horizontal asymptote? Explain its meaning with respect to Zonta's shooting percentage.

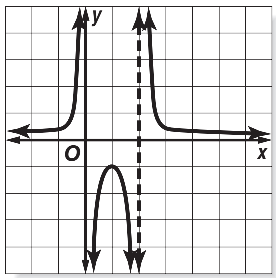
51. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can rational functions be used when buying a group gift?

Include the following in your answer:
- a complete graph of the function $c = \frac{150}{s}$ with asymptotes, and
- an explanation of why only part of the graph is meaningful in the context of the problem.

Standardized Test Practice

52. Which set is the domain of the function graphed at the right?



(A) $\{x \mid x \neq 0, 2\}$

(B) $\{x \mid x \neq -2, 0\}$

(C) $\{x \mid x < 4\}$

(D) $\{x \mid x > -4\}$

53. Which set is the range of the function $y = \frac{x^2 + 8}{2}$?

(A) $\{y \mid y \neq \pm 2\sqrt{2}\}$ (B) $\{y \mid y \geq 4\}$

(C) $\{y \mid y \geq 0\}$ (D) $\{y \mid y \leq 0\}$

Maintain Your Skills

Mixed Review

Simplify each expression. *(Lessons 9-2 and 9-1)*

54. $\frac{3m + 2}{m + n} + \frac{4}{2m + 2n}$ **55.** $\frac{5}{x + 3} - \frac{2}{x - 2}$ **56.** $\frac{2w - 4}{w + 3} \div \frac{2w + 6}{5}$

Find the coordinates of the center and the radius of the circle with the given equation. Then graph the circle. *(Lesson 8-3)*

57. $(x - 6)^2 + (y - 2)^2 = 25$ **58.** $x^2 + y^2 + 4x = 9$

59. **ART** Joyce Jackson purchases works of art for an art gallery. Two years ago, she bought a painting for $20,000, and last year, she bought one for $35,000. If paintings appreciate 14% per year, how much are the two paintings worth now? *(Lesson 7-1)*

Solve each equation by completing the square. *(Lesson 6-4)*

60. $x^2 + 8x + 20 = 0$ **61.** $x^2 + 2x - 120 = 0$ **62.** $x^2 + 7x - 17 = 0$

Getting Ready for the Next Lesson

BASIC SKILL **Solve each proportion.**

63. $\frac{16}{v} = \frac{32}{9}$ **64.** $\frac{7}{25} = \frac{a}{5}$ **65.** $\frac{6}{15} = \frac{8}{s}$ **66.** $\frac{b}{9} = \frac{40}{30}$

CONTENTS

Graphing Calculator Investigation

A Follow-Up of Lesson 9-3

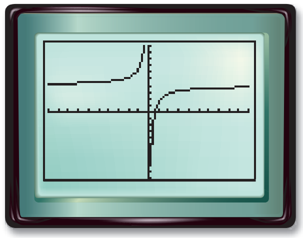
Graphing Rational Functions

A TI-83 Plus graphing calculator can be used to explore the graphs of rational functions. These graphs have some features that never appear in the graphs of polynomial functions.

Example 1 **Graph $y = \frac{8x - 5}{2x}$ in the standard viewing window. Find the equations of any asymptotes.**

- Enter the equation in the Y= list.

KEYSTROKES: [Y=] [(] 8 [X,T,θ,n] [−] 5 [)] [÷] [(] 2 [X,T,θ,n] [)] [ZOOM] 6



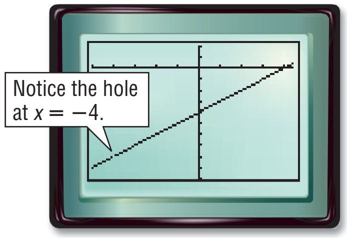
[−10, 10] scl: 1 by [−10, 10] scl: 1

By looking at the equation, we can determine that if $x = 0$, the function is undefined. The equation of the vertical asymptote is $x = 0$. Notice what happens to the y values as x grows larger and as x gets smaller. The y values approach 4. So, the equation for the horizontal asymptote is $y = 4$.

Example 2 **Graph $y = \frac{x^2 - 16}{x + 4}$ in the window [−5, 4.4] by [−10, 2] with scale factors of 1.**

- Because the function is not continuous, put the calculator in dot mode.

KEYSTROKES: [MODE] [▼] [▼] [▼] [▼] [▶] [ENTER]



[−5, 4.4] scl: 1 by [−10, 2] scl: 1

This graph looks like a line with a break in continuity at $x = -4$. This happens because the denominator is 0 when $x = -4$. Therefore, the function is undefined when $x = -4$.

If you TRACE along the graph, when you come to $x = -4$, you will see that there is no corresponding y value.

Exercises

Use a graphing calculator to graph each function. Be sure to show a complete graph. Draw the graph on a sheet of paper. Write the x-coordinates of any points of discontinuity and/or the equations of any asymptotes.

1. $f(x) = \frac{1}{x}$ **2.** $f(x) = \frac{x}{x + 2}$ **3.** $f(x) = \frac{2}{x - 4}$

4. $f(x) = \frac{2x}{3x - 6}$ **5.** $f(x) = \frac{4x + 2}{x - 1}$ **6.** $f(x) = \frac{x^2 - 9}{x + 3}$

7. Which graph(s) has point discontinuity?

8. Describe functions that have point discontinuity.

www.algebra2.com/other_calculator_keystrokes

9-4 Direct, Joint, and Inverse Variation

What You'll Learn

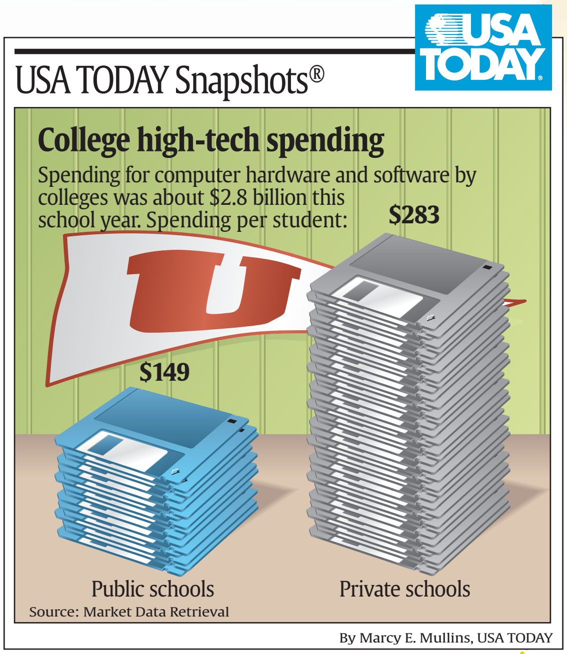
- Recognize and solve direct and joint variation problems.
- Recognize and solve inverse variation problems.

Vocabulary

- direct variation
- constant of variation
- joint variation
- inverse variation

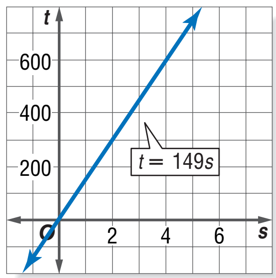
How is variation used to find the total cost given the unit cost?

The total high-tech spending t of an average public college can be found by using the equation $t = 149s$, where s is the number of students.



DIRECT VARIATION AND JOINT VARIATION The relationship given by $t = 149s$ is an example of direct variation. A **direct variation** can be expressed in the form $y = kx$. The k in this equation is a constant and is called the **constant of variation**.

Notice that the graph of $t = 149s$ is a straight line through the origin. An equation of a direct variation is a special case of an equation written in slope-intercept form, $y = mx + b$. When $m = k$ and $b = 0$, $y = mx + b$ becomes $y = kx$. So the slope of a direct variation equation is its constant of variation.



To express a direct variation, we say that y varies directly as x. In other words, as x increases, y increases or decreases at a constant rate.

Key Concept — Direct Variation

y varies directly as x if there is some nonzero constant k such that $y = kx$. k is called the constant of variation.

If you know that y varies directly as x and one set of values, you can use a proportion to find the other set of corresponding values.

$$y_1 = kx_1 \quad \text{and} \quad y_2 = kx_2$$

$$\frac{y_1}{x_1} = k \qquad \frac{y_2}{x_2} = k$$

Therefore, $\frac{y_1}{x_1} = \frac{y_2}{x_2}$.

Using the properties of equality, you can find many other proportions that relate these same x and y values.

Example 1 Direct Variation

If y varies directly as x and $y = 12$ when $x = -3$, find y when $x = 16$.

Use a proportion that relates the values.

$$\frac{y_1}{x_1} = \frac{y_2}{x_2}$$ Direct proportion

$$\frac{12}{-3} = \frac{y_2}{16}$$ $y_1 = 12$, $x_1 = -3$, and $x_2 = 16$

$$16(12) = -3(y_2)$$ Cross multiply.

$$192 = -3y_2$$ Simplify.

$$-64 = y_2$$ Divide each side by -3.

When $x = 16$, the value of y is -64.

Another type of variation is joint variation. **Joint variation** occurs when one quantity varies directly as the product of two or more other quantities.

Key Concept — Joint Variation

y varies jointly as x and z if there is some number k such that $y = kxz$, where $k \neq 0$, $x \neq 0$, and $z \neq 0$.

If you know y varies jointly as x and z and one set of values, you can use a proportion to find the other set of corresponding values.

$$y_1 = kx_1z_1 \quad \text{and} \quad y_2 = kx_2z_2$$

$$\frac{y_1}{x_1z_1} = k \qquad \frac{y_2}{x_2z_2} = k$$

Therefore, $\frac{y_1}{x_1z_1} = \frac{y_2}{x_2z_2}$.

Example 2 Joint Variation

Suppose y varies jointly as x and z. Find y when $x = 8$ and $z = 3$, if $y = 16$ when $z = 2$ and $x = 5$.

Use a proportion that relates the values.

$$\frac{y_1}{x_1z_1} = \frac{y_2}{x_2z_2}$$ Joint variation

$$\frac{16}{5(2)} = \frac{y_2}{8(3)}$$ $y_1 = 16$, $x_1 = 5$, $z_1 = 2$, $x_2 = 8$, and $z_2 = 3$

$$8(3)(16) = 5(2)(y_2)$$ Cross multiply.

$$384 = 10y_2$$ Simplify.

$$38.4 = y_2$$ Divide each side by 10.

When $x = 8$ and $z = 3$, the value of y is 38.4.

INVERSE VARIATION Another type of variation is inverse variation. For two quantities with **inverse variation**, as one quantity increases, the other quantity decreases. For example, speed and time for a fixed distance vary inversely with each other. When you travel to a particular location, as your speed increases, the time it takes to arrive at that location decreases.

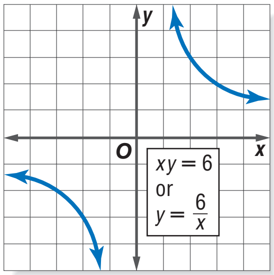
www.algebra2.com/extra_examples

Key Concept — *Inverse Variation*

y varies inversely as x if there is some nonzero constant k such that

$$xy = k \text{ or } y = \frac{k}{x}.$$

Suppose y varies inversely as x such that $xy = 6$ or $y = \frac{6}{x}$. The graph of this equation is shown at the right. Note that in this case, k is a positive value 6, so as the values of x increase, the values of y decrease.



Just as with direct variation and joint variation, a proportion can be used with inverse variation to solve problems where some quantities are known. The following proportion is only one of several that can be formed.

$$x_1y_1 = k \text{ and } x_2y_2 = k$$

$$x_1y_1 = x_2y_2 \quad \text{Substitution Property of Equality}$$

$$\frac{x_1}{y_2} = \frac{x_2}{y_1} \quad \text{Divide each side by } y_1y_2.$$

Example 3 *Inverse Variation*

If r varies inversely as t and $r = 18$ when $t = -3$, find r when $t = -11$.

Use a proportion that relates the values.

$$\frac{r_1}{t_2} = \frac{r_2}{t_1} \quad \text{Inverse variation}$$

$$\frac{18}{-11} = \frac{r_2}{-3} \quad r_1 = 18, t_1 = -3, \text{ and } t_2 = -11$$

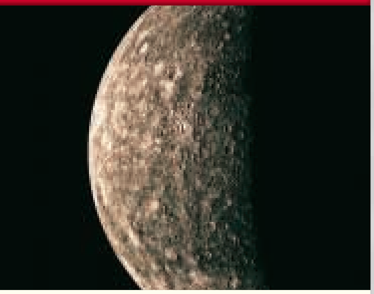
$$18(-3) = -11(r_2) \quad \text{Cross multiply.}$$

$$-54 = -11r_2 \quad \text{Simplify.}$$

$$4\frac{10}{11} = r_2 \quad \text{Divide each side by } -11.$$

When $t = -11$, the value of r is $4\frac{10}{11}$.

More About. . .



Space

Mercury is about 36 million miles from the Sun, making it the closest planet to the Sun. Its proximity to the Sun causes its temperature to be as high as 800°F.

Source: *World Book Encyclopedia*

Example 4 *Use Inverse Variation*

SPACE The apparent length of an object is inversely proportional to one's distance from the object. Earth is about 93 million miles from the Sun. Use the information at the left to find how much larger the diameter of the Sun would appear on Mercury than on Earth.

Explore You know that the apparent diameter of the Sun varies inversely with the distance from the Sun. You also know Mercury's distance from the Sun and Earth's distance from the Sun. You want to determine how much larger the diameter of the Sun appears on Mercury than on Earth.

Plan Let the apparent diameter of the Sun from Earth equal 1 unit and the apparent diameter of the Sun from Mercury equal m. Then use a proportion that relates the values.

Solve

$$\frac{\text{distance from Mercury}}{\text{apparent diameter from Earth}} = \frac{\text{distance from Earth}}{\text{apparent diameter from Mercury}}$$ Inverse variation

$$\frac{36 \text{ million miles}}{1 \text{ unit}} = \frac{93 \text{ million miles}}{m \text{ units}}$$ Substitution

$$(36 \text{ million miles})(m \text{ units}) = (93 \text{ million miles})(1 \text{ unit})$$ Cross multiply.

$$m = \frac{(93 \text{ million miles})(1 \text{ unit})}{36 \text{ million miles}}$$ Divide each side by 36 million miles.

$$m \approx 2.58 \text{ units}$$ Simplify.

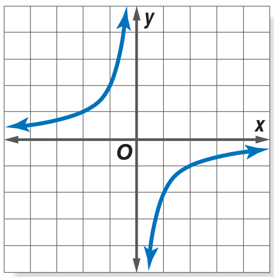
Examine Since distance between the Sun and Earth is between 2 and 3 times the distance between the Sun and Mercury, the answer seems reasonable. From Mercury, the diameter of the Sun will appear about 2.58 times as large as it appears from Earth.

Check for Understanding

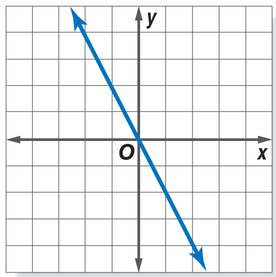
Concept Check

1. **Determine** whether each graph represents a *direct* or an *inverse* variation.

a.



b.



2. **Compare and contrast** $y = 5x$ and $y = -5x$.
3. **OPEN ENDED** Describe two quantities in real life that vary directly with each other and two quantities that vary inversely with each other.

Guided Practice

State whether each equation represents a *direct*, *joint*, or *inverse* variation. Then name the constant of variation.

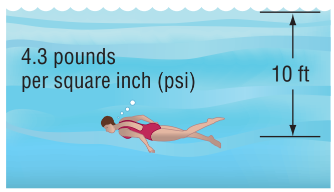
4. $ab = 20$
5. $\frac{y}{x} = -0.5$
6. $A = \frac{1}{2}bh$

Find each value.

7. If y varies directly as x and $y = 18$ when $x = 15$, find y when $x = 20$.
8. Suppose y varies jointly as x and z. Find y when $x = 9$ and $z = -5$, if $y = -90$ when $z = 15$ and $x = -6$.
9. If y varies inversely as x and $y = -14$ when $x = 12$, find x when $y = 21$.

Application

SWIMMING **For Exercises 10–13, use the following information.**
When a person swims underwater, the pressure in his or her ears varies directly with the depth at which he or she is swimming.



10. Write an equation of direct variation that represents this situation.
11. Find the pressure at 60 feet.
12. It is unsafe for amateur divers to swim where the water pressure is more than 65 pounds per square inch. How deep can an amateur diver safely swim?
13. Make a table showing the number of pounds of pressure at various depths of water. Use the data to draw a graph of pressure versus depth.

Practice and Apply

Homework Help

Exercises	Examples
14–37	1–3
38–53	4

Extra Practice

See page 848.

State whether each equation represents a *direct*, *joint*, or *inverse* variation. Then name the constant of variation.

14. $\frac{n}{m} = 1.5$ **15.** $a = 5bc$ **16.** $vw = -18$ **17.** $3 = \frac{a}{b}$

18. $p = \frac{12}{q}$ **19.** $y = -7x$ **20.** $V = \frac{1}{3}Bh$ **21.** $\frac{2.5}{t} = s$

22. CHEMISTRY Boyle's Law states that when a sample of gas is kept at a constant temperature, the volume varies inversely with the pressure exerted on it. Write an equation for Boyle's Law that expresses the variation in volume V as a function of pressure P.

23. CHEMISTRY Charles' Law states that when a sample of gas is kept at a constant pressure, its volume V will increase as the temperature t increases. Write an equation for Charles' Law that expresses volume as a function.

24. GEOMETRY How does the circumference of a circle vary with respect to its radius? What is the constant of variation?

25. TRAVEL A map is scaled so that 3 centimeters represents 45 kilometers. How far apart are two towns if they are 7.9 centimeters apart on the map?

Career Choices

Travel Agent

Travel agents give advice and make arrangements for transportation, accommodations, and recreation. For international travel, they also provide information on customs and currency exchange.

Online Research

For information about a career as a travel agent, visit: www.algebra2.com/careers

Find each value.

26. If y varies directly as x and $y = 15$ when $x = 3$, find y when $x = 12$.

27. If y varies directly as x and $y = 8$ when $x = 6$, find y when $x = 15$.

28. Suppose y varies jointly as x and z. Find y when $x = 2$ and $z = 27$, if $y = 192$ when $x = 8$ and $z = 6$.

29. If y varies jointly as x and z and $y = 80$ when $x = 5$ and $z = 8$, find y when $x = 16$ and $z = 2$.

30. If y varies inversely as x and $y = 5$ when $x = 10$, find y when $x = 2$.

31. If y varies inversely as x and $y = 16$ when $x = 5$, find y when $x = 20$.

32. If y varies inversely as x and $y = 2$ when $x = 25$, find x when $y = 40$.

33. If y varies inversely as x and $y = 4$ when $x = 12$, find y when $x = 5$.

34. If y varies directly as x and $y = 9$ when x is -15, find y when $x = 21$.

35. If y varies directly as x and $x = 6$ when $y = 0.5$, find y when $x = 10$.

36. Suppose y varies jointly as x and z. Find y when $x = \frac{1}{2}$ and $z = 6$, if $y = 45$ when $x = 6$ and $z = 10$.

37. If y varies jointly as x and z and $y = \frac{1}{8}$ when $x = \frac{1}{2}$ and $z = 3$, find y when $x = 6$ and $z = \frac{1}{3}$.

38. WORK Paul drove from his house to work at an average speed of 40 miles per hour. The drive took him 15 minutes. If the drive home took him 20 minutes and he used the same route in reverse, what was his average speed going home?

39. WATER SUPPLY Many areas of Northern California depend on the snowpack of the Sierra Nevada Mountains for their water supply. If 250 cubic centimeters of snow will melt to 28 cubic centimeters of water, how much water does 900 cubic centimeters of snow produce?

40. **RESEARCH** According to Johannes Kepler's third law of planetary motion, the ratio of the square of a planet's period of revolution around the Sun to the cube of its mean distance from the Sun is constant for all planets. Verify that this is true for at least three planets.

More About. . .

Biology

In order to sustain itself in its cold habitat, a Siberian tiger requires 20 pounds of meat per day.

Source: *Wildlife Fact File*

BIOLOGY **For Exercises 41–43, use the information at the left.**

41. Write an equation to represent the amount of meat needed to sustain s Siberian tigers for d days.

42. Is your equation in Exercise 41 a *direct*, *joint*, or *inverse* variation?

43. How much meat do three Siberian tigers need for the month of January?

LAUGHTER **For Exercises 44–46, use the following information.**
According to *The Columbus Dispatch*, the average American laughs 15 times per day.

44. Write an equation to represent the average number of laughs produced by m household members during a period of d days.

45. Is your equation in Exercise 44 a *direct*, *joint*, or *inverse* variation?

46. Assume that members of your household laugh the same number of times each day as the average American. How many times would the members of your household laugh in a week?

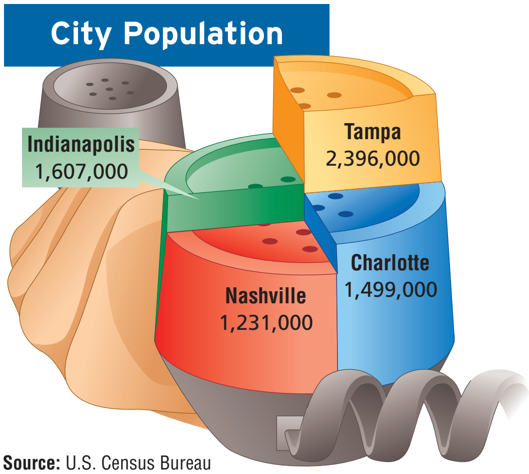
ARCHITECTURE **For Exercises 47–49, use the following information.**
When designing buildings such as theaters, auditoriums, or museums architects have to consider how sound travels. Sound intensity I is inversely proportional to the square of the distance from the sound source d.

47. Write an equation that represents this situation.

48. If d is the independent variable and I is the dependent variable, graph the equation from Exercise 47 when $k = 16$.

49. If a person in a theater moves to a seat twice as far from the speakers, compare the new sound intensity to that of the original.

TELECOMMUNICATIONS **For Exercises 50–53, use the following information.**
It has been found that the average number of daily phone calls C between two cities is directly proportional to the product of the populations P_1 and P_2 of two cities and inversely proportional to the square of the distance d between the cities. That is, $C = \frac{kP_1P_2}{d^2}$.



Source: U.S. Census Bureau

50. The distance between Nashville and Charlotte is about 425 miles. If the average number of daily phone calls between the cities is 204,000, find the value of k and write the equation of variation. Round to the nearest hundredth.

51. Nashville is about 680 miles from Tampa. Find the average number of daily phone calls between them.

52. The average daily phone calls between Indianapolis and Charlotte is 133,380. Find the distance between Indianapolis and Charlotte.

53. Could you use this formula to find the populations or the average number of phone calls between two adjoining cities? Explain.

54. **CRITICAL THINKING** Write a real-world problem that involves a joint variation. Solve the problem.

55. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is variation used to find the total cost given the unit cost?

Include the following in your answer:

- an explanation of why the equation for the total cost is a direct variation, and
- a problem involving unit cost and total cost of an item and its solution.

Standardized Test Practice

56. If the ratio of $2a$ to $3b$ is 4 to 5, what is the ratio of $5a$ to $4b$?

(A) $\frac{4}{3}$ (B) $\frac{3}{4}$ (C) $\frac{9}{8}$ (D) $\frac{3}{2}$

57. Suppose b varies inversely as the square of a. If a is multiplied by 9, which of the following is true for the value of b?

(A) It is multiplied by $\frac{1}{3}$.
(B) It is multiplied by $\frac{1}{9}$.
(C) It is multiplied by $\frac{1}{81}$.
(D) It is multiplied by 3.

Maintain Your Skills

Mixed Review

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function. *(Lesson 9-3)*

58. $f(x) = \frac{x+1}{x^2-1}$
59. $f(x) = \frac{x+3}{x^2+x-12}$
60. $f(x) = \frac{x^2+4x+3}{x+3}$

Simplify each expression. *(Lesson 9-2)*

61. $\frac{3x}{x-y} + \frac{4x}{y-x}$
62. $\frac{t}{t+2} - \frac{2}{t^2-4}$
63. $\frac{m - \frac{1}{m}}{1 + \frac{4}{m} - \frac{5}{m^2}}$

64. **ASTRONOMY** The distance from Earth to the Sun is approximately 93,000,000 miles. Write this number in scientific notation. *(Lesson 5-1)*

State the slope and the y-intercept of the graph of each equation. *(Lesson 2-4)*

65. $y = 0.4x + 1.2$
66. $2y = 6x + 14$
67. $3x + 5y = 15$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Identify each function as S for step, C for constant, A for absolute value, or P for piecewise.** *(To review **special functions**, see Lesson 2-6.)*

68. $h(x) = \frac{2}{3}$
69. $g(x) = 3|x|$
70. $f(x) = [\![2x]\!]$
71. $f(x) = \begin{cases} 1 \text{ if } x > 0 \\ -1 \text{ if } x \le 0 \end{cases}$
72. $h(x) = |x - 2|$
73. $g(x) = -3$

Practice Quiz 2

Lessons 9-3 and 9-4

Graph each rational function. *(Lesson 9-3)*

1. $f(x) = \frac{x-1}{x-4}$
2. $f(x) = \frac{-2}{x^2-6x+9}$

Find each value. *(Lesson 9-4)*

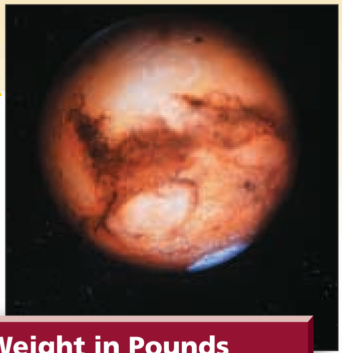
3. If y varies inversely as x and $x = 14$ when $y = 7$, find x when $y = 2$.
4. If y varies directly as x and $y = 1$ when $x = 5$, find y when $x = 22$.
5. If y varies jointly as x and z and $y = 80$ when $x = 25$ and $z = 4$, find y when $x = 20$ and $z = 7$.

9-5 Classes of Functions

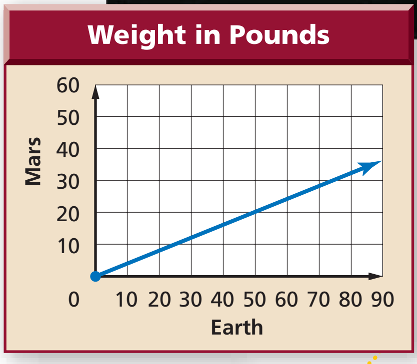
What You'll Learn

- Identify graphs as different types of functions.
- Identify equations as different types of functions.

How can graphs of functions be used to determine a person's weight on a different planet?



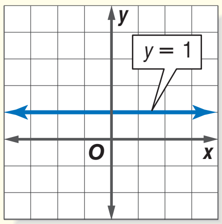
The purpose of the 2001 Mars Odyssey Mission is to study conditions on Mars. The findings will help NASA prepare for a possible mission with human explorers. The graph at the right compares a person's weight on Earth with his or her weight on Mars. This graph represents a direct variation, which you studied in the previous lesson.



IDENTIFY GRAPHS In this book, you have studied several types of graphs representing special functions. The following is a summary of these graphs.

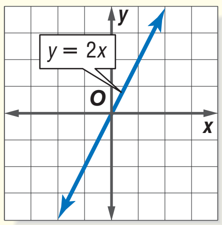
Concept Summary — *Special Functions*

Constant Function



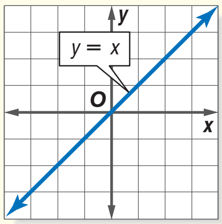
The general equation of a constant function is $y = a$, where a is any number. Its graph is a horizontal line that crosses the y-axis at a.

Direct Variation Function



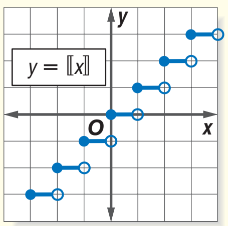
The general equation of a direct variation function is $y = ax$, where a is a nonzero constant. Its graph is a line that passes through the origin and is neither horizontal nor vertical.

Identity Function



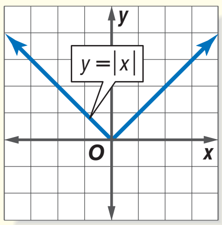
The identity function $y = x$ is a special case of the direct variation function in which the constant is 1. Its graph passes through all points with coordinates (a, a).

Greatest Integer Function



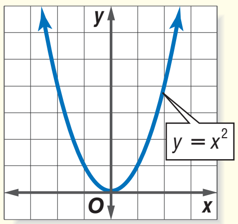
If an equation includes an expression inside the greatest integer symbol, the function is a greatest integer function. Its graph looks like steps.

Absolute Value Function



An equation with a direct variation expression inside absolute value symbols is an absolute value function. Its graph is in the shape of a V.

Quadratic Function

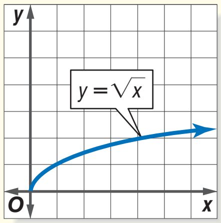


The general equation of a quadratic function is $y = ax^2 + bx + c$, where $a \neq 0$. Its graph is a parabola.

(continued on the next page)

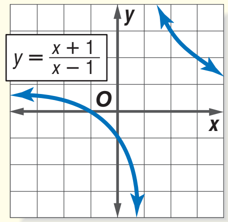
Concept Summary — Special Functions

Square Root Function



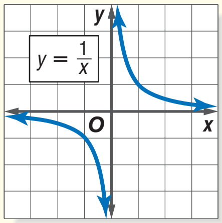
If an equation includes an expression inside the radical sign, the function is a square root function. Its graph is a curve that starts at a point and continues in only one direction.

Rational Function



The general equation for a rational function is $y = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial functions. Its graph has one or more asymptotes and/or holes.

Inverse Variation Function

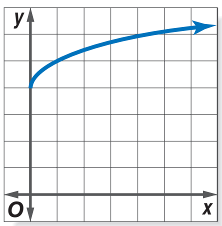


The inverse variation function $y = \frac{a}{x}$ is a special case of the rational function where $p(x)$ is a constant and $q(x) = x$. Its graph has two asymptotes, $x = 0$ and $y = 0$.

Example 1 Identify a Function Given the Graph

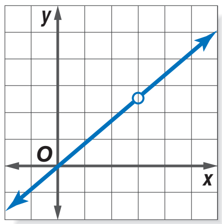
Identify the type of function represented by each graph.

a.



The graph has a starting point and curves in one direction. The graph represents a square root function.

b.



The graph appears to be a direct variation since it is a straight line passing through the origin. However, the hole indicates that it represents a rational function.

More About. . .

Rockerty

A rocket-powered airplane called the *X-15* set an altitude record for airplanes by flying 67 miles above Earth.

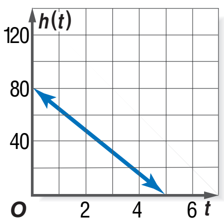
Source: *World Book Encyclopedia*

IDENTIFY EQUATIONS If you can identify an equation as a type of function, you can determine the shape of the graph.

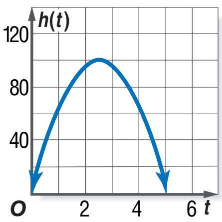
Example 2 Match Equation with Graph

ROCKETRY Emily launched a toy rocket from ground level. The height above the ground level h, in feet, after t seconds is given by the formula $h(t) = -16t^2 + 80t$. Which graph depicts the height of the rocket during its flight?

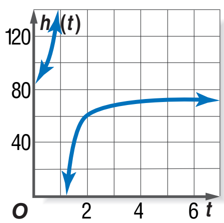
a.



b.



c.



The function includes a second-degree polynomial. Therefore, it is a quadratic function, and its graph is a parabola. Graph b is the only parabola. Therefore, the answer is graph **b**.

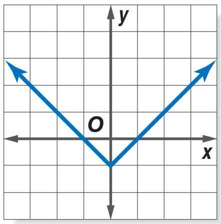
Sometimes recognizing an equation as a specific type of function can help you graph the function.

Example 3 Identify a Function Given its Equation

Identify the type of function represented by each equation. Then graph the equation.

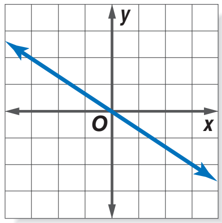
a. $y = |x| - 1$

Since the equation includes an expression inside absolute value symbols, it is an absolute value function. Therefore, the graph will be in the shape of a V. Determine some points on the graph and use what you know about graphs of absolute value functions to graph the function.



b. $y = -\frac{2}{3}x$

The function is in the form $y = ax$, where $a = -\frac{2}{3}$. Therefore, it is a direct variation function. The graph passes through the origin and has a slope of $-\frac{2}{3}$.



Check for Understanding

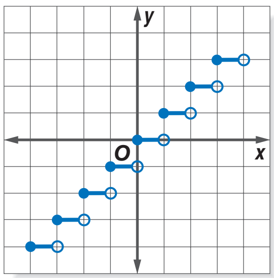
Concept Check

1. **OPEN ENDED** Find a counterexample to the statement *All functions are continuous.* Describe your function.
2. **Name** three special functions whose graphs are straight lines. Give an example of each function.
3. **Describe** the graph of $y = [\![x + 2]\!]$.

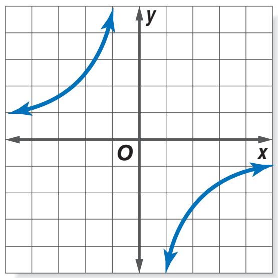
Guided Practice

Identify the type of function represented by each graph.

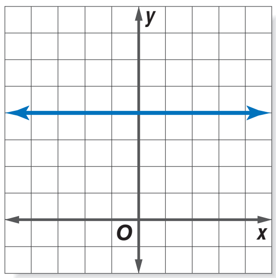
4.



5.

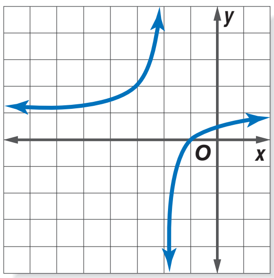


6.

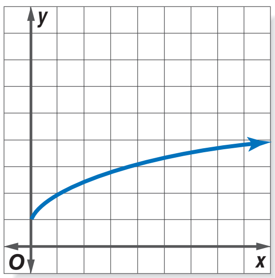


Match each graph with an equation at the right.

7.



8.



a. $y = x^2 + 2x + 3$
b. $y = \sqrt{x} + 1$
c. $y = \frac{x + 1}{x + 2}$
d. $y = [\![2x]\!]$

Identify the type of function represented by each equation. Then graph the equation.

9. $y = x$ **10.** $y = -x^2 + 2$ **11.** $y = |x + 2|$

Application

12. GEOMETRY Write the equation for the area of a circle. Identify the equation as a type of function. Describe the graph of the function.

Practice and Apply

Homework Help

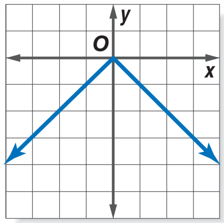
For Exercises	See Examples
13–18	1
19–22, 31–36	2
23–30	3

Extra Practice

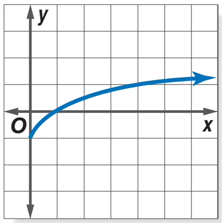
See page 848.

Identify the type of function represented by each graph.

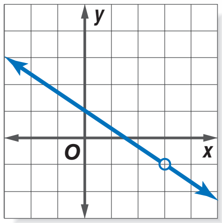
13.



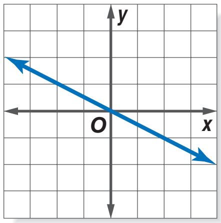
14.



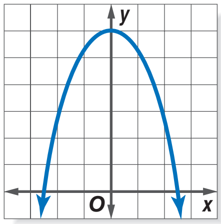
15.



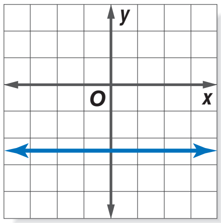
16.



17.



18.

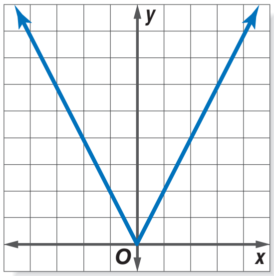


WebQuest

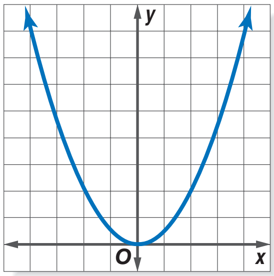
You can use functions to determine the relationship between primary and secondary earthquake waves. Visit www.algebra2.com/webquest to continue work on your WebQuest project.

Match each graph with an equation at the right.

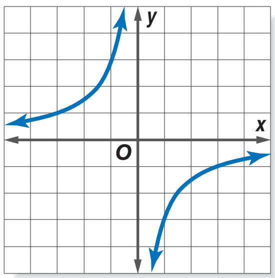
19.



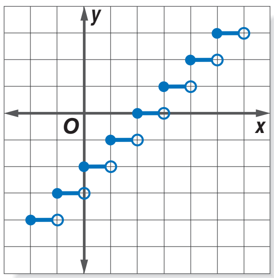
20.



21.



22.



a. $y = [\![x]\!] - 2$

b. $y = 2|x|$

c. $y = 2\sqrt{x}$

d. $y = -3x$

e. $y = 0.5x^2$

f. $y = -\dfrac{3}{x + 1}$

g. $y = -\dfrac{3}{x}$

Identify the type of function represented by each equation. Then graph the equation.

23. $y = -1.5$ **24.** $y = 2.5x$ **25.** $y = \sqrt{9x}$ **26.** $y = \dfrac{4}{x}$

27. $y = \dfrac{x^2 - 1}{x - 1}$ **28.** $y = 3[\![x]\!]$ **29.** $y = |2x|$ **30.** $y = 2x^2$

HEALTH **For Exercises 31–33, use the following information.**
A woman painting a room will burn an average of 4.5 Calories per minute.

31. Write an equation for the number of Calories burned in m minutes.

32. Identify the equation in Exercise 31 as a type of function.

33. Describe the graph of the function.

More About. . .

Architecture

The Gateway Arch is 630 feet high and is the tallest monument in the United States.

Source: *World Book Encyclopedia*

34. **ARCHITECTURE** The shape of the Gateway Arch of the Jefferson National Expansion Memorial in St. Louis, Missouri, resembles the graph of the function $f(x) = -0.00635x^2 + 4.0005x - 0.07875$, where x is in feet. Describe the shape of the Gateway Arch.

MAIL **For Exercises 35 and 36, use the following information.**
In 2001, the cost to mail a first-class letter was 34¢ for any weight up to and including 1 ounce. Each additional ounce or part of an ounce added 21¢ to the cost.

35. Make a graph showing the postal rates to mail any letter from 0 to 8 ounces.

36. Compare your graph in Exercise 35 to the graph of the greatest integer function.

37. **CRITICAL THINKING** Identify each table of values as a type of function.

a.

x	$f(x)$
−5	7
−3	5
−1	3
0	2
1	3
3	5
5	7
7	9

b.

x	$f(x)$
−5	24
−3	8
−1	0
0	−1
1	0
3	8
5	24
7	48

c.

x	$f(x)$
−1.3	−1
−1.7	−1
0	1
0.8	1
0.9	1
1	2
1.5	2
2.3	3

d.

x	$f(x)$
−5	undefined
−3	undefined
−1	undefined
0	0
1	1
4	2
9	3
16	4

38. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can graphs of functions be used to determine a person's weight on a different planet?

Include the following in your answer:

- an explanation of why the graph comparing weight on Earth and Mars represents a direct variation function, and
- an equation and a graph comparing a person's weight on Earth and Venus if a person's weight on Venus is 0.9 of his or her weight on Earth.

Standardized Test Practice

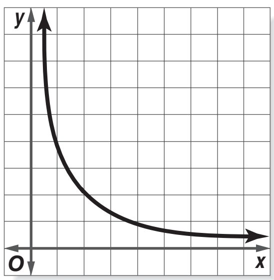
39. The curve at the right could be part of the graph of which function?

Ⓐ $y = \sqrt{x}$

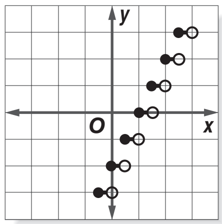
Ⓑ $y = x^2 - 5x + 4$

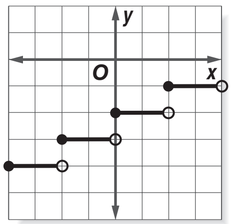
Ⓒ $xy = 4$

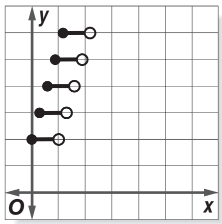
Ⓓ $y = -x + 20$

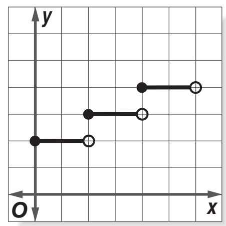


40. If $g(x) = [\![x]\!]$, which of the following is the graph of $g\left(\frac{x}{2}\right) + 2$?

(A)


(B)


(C)


(D)


Maintain Your Skills

Mixed Review

41. If x varies directly as y and $y = \frac{1}{5}$ when $x = 11$, find x when $y = \frac{2}{5}$. *(Lesson 9–4)*

Graph each rational function. *(Lesson 9-3)*

42. $f(x) = \dfrac{3}{x+2}$

43. $f(x) = \dfrac{8}{(x-1)(x+3)}$

44. $f(x) = \dfrac{x^2 - 5x + 4}{x - 4}$

Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola. *(Lesson 8-2)*

45. $\frac{1}{2}(y + 1) = (x - 8)^2$

46. $x = \frac{1}{4}y^2 - \frac{1}{2}y - 3$

47. $3x - y^2 = 8y + 31$

Find each product, if possible. *(Lesson 4-3)*

48. $\begin{bmatrix} 3 & -5 \\ 2 & 7 \end{bmatrix} \cdot \begin{bmatrix} 5 & 1 & -3 \\ 8 & -4 & 9 \end{bmatrix}$

49. $\begin{bmatrix} 4 & -1 & 6 \\ 1 & 5 & -8 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 9 & -6 \end{bmatrix}$

Solve each system of equations by using either substitution or elimination. *(Lesson 3-2)*

50. $3x + 5y = -4$
$2x - 3y = 29$

51. $3a - 2b = -3$
$3a + b = 3$

52. $3s - 2t = 10$
$4s + t = 6$

Determine the value of r so that a line through the points with the given coordinates has the given slope. *(Lesson 2-3)*

53. $(r, 2), (4, -6)$; slope $= \frac{-8}{3}$

54. $(r, 6), (8, 4)$; slope $= \frac{1}{2}$

55. Evaluate $[(-7 + 4) \times 5 - 2] \div 6$. *(Lesson 1-1)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find the LCM of each set of polynomials.**
*(To review **least common multiples of polynomials,** see Lesson 9-2.)*

56. $15ab^2c, 6a^3, 4bc^2$

57. $9x^3, 5xy^2, 15x^2y^3$

58. $5d - 10, 3d - 6$

59. $x^2 - y^2, 3x + 3y$

60. $a^2 - 2a - 3, a^2 - a - 6$

61. $2t^2 - 9t - 5, t^2 + t - 30$

9-6 Solving Rational Equations and Inequalities

What You'll Learn

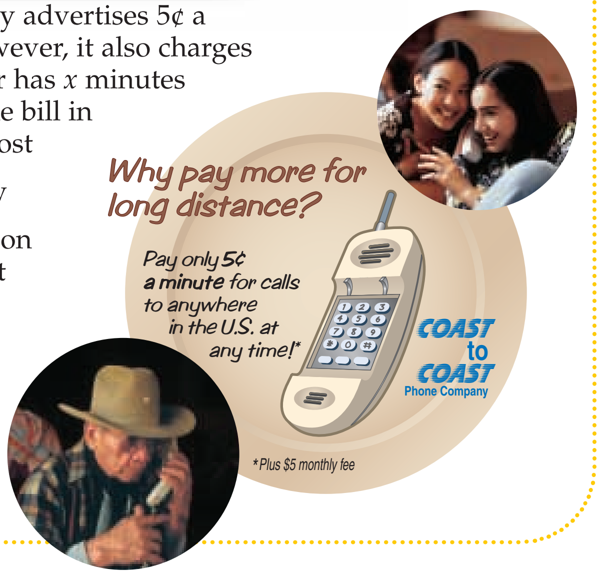
- Solve rational equations.
- Solve rational inequalities.

How are rational equations used to solve problems involving unit price?

Vocabulary

- rational equation
- rational inequality

The Coast to Coast Phone Company advertises 5¢ a minute for long-distance calls. However, it also charges a monthly fee of \$5. If the customer has x minutes in long distance calls last month, the bill in cents will be $500 + 5x$. The actual cost per minute is $\frac{500 + 5x}{x}$. To find how many long-distance minutes a person would need to make the actual cost per minute 6¢, you would need to solve the equation $\frac{500 + 5x}{x} = 6$.



SOLVE RATIONAL EQUATIONS The equation $\frac{500 + 5x}{x} = 6$ is an example of a rational equation. In general, any equation that contains one or more rational expressions is called a **rational equation**.

Rational equations are easier to solve if the fractions are eliminated. You can eliminate the fractions by multiplying each side of the equation by the least common denominator (LCD). Remember that when you multiply each side by the LCD, each term on each side must be multiplied by the LCD.

Example 1 Solve a Rational Equation

Solve $\frac{9}{28} + \frac{3}{z + 2} = \frac{3}{4}$. Check your solution.

The LCD for the three denominators is $28(z + 2)$.

$$\frac{9}{28} + \frac{3}{z + 2} = \frac{3}{4} \quad \text{Original equation}$$

$$28(z + 2)\left(\frac{9}{28} + \frac{3}{z + 2}\right) = 28(z + 2)\left(\frac{3}{4}\right) \quad \text{Multiply each side by } 28(z + 2).$$

$$\overset{1}{\cancel{28}}(z + 2)\left(\frac{9}{\underset{1}{\cancel{28}}}\right) + 28\overset{1}{\cancel{(z + 2)}}\left(\frac{3}{\underset{1}{\cancel{z + 2}}}\right) = \overset{7}{\cancel{28}}(z + 2)\left(\frac{3}{\underset{1}{\cancel{4}}}\right) \quad \text{Distributive Property}$$

$$(9z + 18) + 84 = 21z + 42 \quad \text{Simplify.}$$

$$9z + 102 = 21z + 42 \quad \text{Simplify.}$$

$$60 = 12z \quad \text{Subtract } 9z \text{ and } 42 \text{ from each side.}$$

$$5 = z \quad \text{Divide each side by 12.}$$

CHECK

$$\frac{9}{28} + \frac{3}{z+2} = \frac{3}{4}$$ Original equation

$$\frac{9}{28} + \frac{3}{5+2} \stackrel{?}{=} \frac{3}{4}$$ $z = 5$

$$\frac{9}{28} + \frac{3}{7} \stackrel{?}{=} \frac{3}{4}$$ Simplify.

$$\frac{9}{28} + \frac{12}{28} \stackrel{?}{=} \frac{3}{4}$$ Simplify.

$$\frac{3}{4} = \frac{3}{4} \checkmark$$ The solution is correct.

The solution is 5.

When solving a rational equation, any possible solution that results in a zero in the denominator must be excluded from your list of solutions.

Example 2 Elimination of a Possible Solution

Solve $r + \frac{r^2-5}{r^2-1} = \frac{r^2+r+2}{r+1}$. Check your solution.

The LCD is $(r^2 - 1)$.

$$r + \frac{r^2-5}{r^2-1} = \frac{r^2+r+2}{r+1}$$ Original equation

$$(r^2-1)\left(r + \frac{r^2-5}{r^2-1}\right) = (r^2-1)\left(\frac{r^2+r+2}{r+1}\right)$$ Multiply each side by the LCD, $(r^2 - 1)$.

$$(r^2-1)r + \overset{1}{\cancel{(r^2-1)}}\left(\frac{r^2-5}{\underset{1}{\cancel{r^2-1}}}\right) = \overset{(r-1)}{\cancel{(r^2-1)}}\left(\frac{r^2+r+2}{\underset{1}{\cancel{r+1}}}\right)$$ Distributive Property

$$(r^3 - r) + (r^2 - 5) = (r-1)(r^2+r+2)$$ Simplify.

$$r^3 + r^2 - r - 5 = r^3 + r - 2$$ Simplify.

$$r^2 - 2r - 3 = 0$$ Subtract $(r^3 + r - 2)$ from each side.

$$(r-3)(r+1) = 0$$ Factor.

$$r - 3 = 0 \quad \text{or} \quad r + 1 = 0$$ Zero Product Property

$$r = 3 \qquad\qquad r = -1$$

CHECK

$$r + \frac{r^2-5}{r^2-1} = \frac{r^2+r+2}{r+1}$$ Original equation

$$3 + \frac{3^2-5}{3^2-1} \stackrel{?}{=} \frac{3^2+3+2}{3+1}$$ $r = 3$

$$3 + \frac{4}{8} \stackrel{?}{=} \frac{14}{4}$$ Simplify.

$$\frac{7}{2} = \frac{7}{2} \checkmark$$

$$r + \frac{r^2-5}{r^2-1} = \frac{r^2+r+2}{r+1}$$ Original equation

$$-1 + \frac{(-1)^2-5}{(-1)^2-1} \stackrel{?}{=} \frac{(-1)^2+(-1)+2}{-1+1}$$ $r = -1$

$$-1 + \frac{-4}{0} \stackrel{?}{=} \frac{2}{0}$$ Simplify.

Since $r = -1$ results in a zero in the denominator, eliminate -1 from the list of solutions.

The solution is 3.

Study Tip

Extraneous Solutions

Multiplying each side of an equation by the LCD of rational expressions can yield results that are not solutions of the original equation. These solutions are called *extraneous solutions.*

Some real-world problems can be solved with rational equations.

Example 3 Work Problem

TUNNELS **When building the Chunnel, the English and French each started drilling on opposite sides of the English Channel. The two sections became one in 1990. The French used more advanced drilling machinery than the English. Suppose the English could drill the Chunnel in 6.2 years and the French could drill it in 5.8 years. How long would it have taken the two countries to drill the tunnel?**

In 1 year, the English could complete $\frac{1}{6.2}$ of the tunnel.

In 2 years, the English could complete $\frac{1}{6.2} \cdot 2$ or $\frac{2}{6.2}$ of the tunnel.

In t years, the English could complete $\frac{1}{6.2} \cdot t$ or $\frac{t}{6.2}$ of the tunnel.

Likewise, in t years, the French could complete $\frac{1}{5.8} \cdot t$ or $\frac{t}{5.8}$ of the tunnel.

Together, they completed the whole tunnel.

Part completed by the English	plus	part completed by the French	equals	entire tunnel.
$\frac{t}{6.2}$	$+$	$\frac{t}{5.8}$	$=$	1

Solve the equation.

$\frac{t}{6.2} + \frac{t}{5.8} = 1$	Original equation
$17.98\left(\frac{t}{6.2} + \frac{t}{5.8}\right) = 17.98(1)$	Multiply each side by 17.98.
$17.98\left(\frac{t}{6.2}\right) + 17.98\left(\frac{t}{5.8}\right) = 17.98$	Distributive Property
$2.9t + 3.1t = 17.98$	Simplify.
$6t = 17.98$	Simplify.
$t \approx 3.00$	Divide each side by 6.

It would have taken about 3 years to build the Chunnel.

More About. . .

Tunnels

The Chunnel is a tunnel under the English Channel that connects England with France. It is 32 miles long with 23 miles of the tunnel under water.

Source: www.pbs.org

Rate problems frequently involve rational equations.

Example 4 Rate Problem

NAVIGATION **The speed of the current in the Puget sound is 5 miles per hour. A barge travels 26 miles with the current and returns in $10\frac{2}{3}$ hours. What is the speed of the barge in still water?**

WORDS The formula that relates distance, time, and rate is $d = rt$ or $\frac{d}{r} = t$.

VARIABLES Let r be the speed of the barge in still water. Then the speed of the barge with the current is $r + 5$, and the speed of the barge against the current is $r - 5$.

	Time going with the current	plus	time going against the current	equals	total time.
EQUATION	$\frac{26}{r + 5}$	$+$	$\frac{26}{r - 5}$	$=$	$10\frac{2}{3}$

(continued on the next page)

Solve the equation.

$$\frac{26}{r+5} + \frac{26}{r-5} = 10\frac{2}{3}$$ Original equation

$$3(r^2-25)\left(\frac{26}{r+5} + \frac{26}{r-5}\right) = 3(r^2-25)\left(10\frac{2}{3}\right)$$ Multiply each side by $3(r^2 - 25)$.

$$3(r^2-25)\left(\frac{26}{r+5}\right) + 3(r^2-25)\left(\frac{26}{r-5}\right) = 3(r^2-25)\left(\frac{32}{3}\right)$$ Distributive Property

$$(78r - 390) + (78r + 390) = 32r^2 - 800$$ Simplify.

$$156r = 32r^2 - 800$$ Simplify.

$$0 = 32r^2 - 156r - 800$$ Subtract $156r$ from each side.

$$0 = 8r^2 - 39r - 200$$ Divide each side by 4.

Study Tip

Look Back

To review the **Quadratic Formula,** see Lesson 6-5.

Use the Quadratic Formula to solve for r.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$r = \frac{-(-39) \pm \sqrt{(-39)^2 - 4(8)(-200)}}{2(8)}$$ $x = r$, $a = 8$, $b = -39$, and $c = -200$

$$r = \frac{39 \pm \sqrt{7921}}{16}$$ Simplify.

$$r = \frac{39 \pm 89}{16}$$ Simplify.

$$r = 8 \text{ or } -3.125$$ Simplify.

Since the speed must be positive, the answer is 8 miles per hour.

SOLVE RATIONAL INEQUALITIES

Inequalities that contain one or more rational expressions are called **rational inequalities**. To solve rational inequalities, complete the following steps.

Step 1 State the excluded values.

Step 2 Solve the related equation.

Step 3 Use the values determined in Steps 1 and 2 to divide a number line into regions. Test a value in each region to determine which regions satisfy the original inequality.

Example 5 Solve a Rational Inequality

Solve $\frac{1}{4a} + \frac{5}{8a} > \frac{1}{2}$.

Step 1 Values that make a denominator equal to 0 are excluded from the domain. For this inequality, the excluded value is 0.

Step 2 Solve the related equation.

$$\frac{1}{4a} + \frac{5}{8a} = \frac{1}{2}$$ Related equation

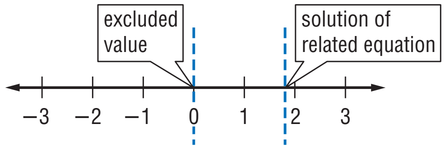
$$8a\left(\frac{1}{4a} + \frac{5}{8a}\right) = 8a\left(\frac{1}{2}\right)$$ Multiply each side by $8a$.

$$2 + 5 = 4a$$ Simplify.

$$7 = 4a$$ Add.

$$1\frac{3}{4} = a$$ Divide each side by 4.

Step 3 Draw vertical lines at the excluded value and at the solution to separate the number line into regions.



Now test a sample value in each region to determine if the values in the region satisfy the inequality.

Test $a = -1$.

$$\frac{1}{4(-1)} + \frac{5}{8(-1)} \stackrel{?}{>} \frac{1}{2}$$
$$-\frac{1}{4} - \frac{5}{8} \stackrel{?}{>} \frac{1}{2}$$
$$-\frac{7}{8} \not> \frac{1}{2}$$

$a < 0$ is *not* a solution.

Test $a = 1$.

$$\frac{1}{4(1)} + \frac{5}{8(1)} \stackrel{?}{>} \frac{1}{2}$$
$$\frac{1}{4} + \frac{5}{8} \stackrel{?}{>} \frac{1}{2}$$
$$\frac{7}{8} > \frac{1}{2} \quad \checkmark$$

$0 < a < 1\frac{3}{4}$ is a solution.

Test $a = 2$.

$$\frac{1}{4(2)} + \frac{5}{8(2)} \stackrel{?}{>} \frac{1}{2}$$
$$\frac{1}{8} + \frac{5}{16} \stackrel{?}{>} \frac{1}{2}$$
$$\frac{7}{16} \not> \frac{1}{2}$$

$a > 1\frac{3}{4}$ is *not* a solution.

The solution is $0 < a < 1\frac{3}{4}$.

Check for Understanding

Concept Check

1. **OPEN ENDED** Write a rational equation that can be solved by first multiplying each side by $5(a + 2)$.

2. State the number by which you would multiply each side of $\frac{x}{x+4} + \frac{1}{2} = 1$ in order to solve the equation. What value(s) of x cannot be a solution?

3. **FIND THE ERROR** Jeff and Dustin are solving $2 - \frac{3}{a} = \frac{2}{3}$.

Jeff	Dustin
$2 - \frac{3}{a} = \frac{2}{3}$	$2 - \frac{3}{a} = \frac{2}{3}$
$6a - 9 = 2a$	$2 - 9 = 2a$
$4a = 9$	$-7 = 2a$
$a = 2.25$	$-3.5 = a$

Who is correct? Explain your reasoning.

Guided Practice

Solve each equation or inequality. Check your solutions.

4. $\frac{2}{d} + \frac{1}{4} = \frac{11}{12}$

5. $t + \frac{12}{t} - 8 = 0$

6. $\frac{1}{x-1} + \frac{2}{x} = 0$

7. $\frac{12}{v^2 - 16} - \frac{24}{v - 4} = 3$

8. $\frac{4}{c+2} > 1$

9. $\frac{1}{3v} + \frac{1}{4v} < \frac{1}{2}$

Application

10. **WORK** A bricklayer can build a wall of a certain size in 5 hours. Another bricklayer can do the same job in 4 hours. If the bricklayers work together, how long would it take to do the job?

Practice and Apply

Homework Help

For Exercises	See Examples
11–30	1, 2, 5
31–39	3, 4

Extra Practice

See page 849.

Solve each equation or inequality. Check your solutions.

11. $\frac{y}{y+1} = \frac{2}{3}$

12. $\frac{p}{p-2} = \frac{2}{5}$

13. $s + 5 = \frac{6}{s}$

14. $a + 1 = \frac{6}{a}$

15. $\frac{7}{a+1} > 7$

16. $\frac{10}{m+1} > 5$

17. $\frac{9}{t-3} = \frac{t-4}{t-3} + \frac{1}{4}$

18. $\frac{w}{w-1} + w = \frac{4w-3}{w-1}$

19. $5 + \frac{1}{t} > \frac{16}{t}$

20. $7 - \frac{2}{b} < \frac{5}{b}$

21. $\frac{2}{3y} + \frac{5}{6y} > \frac{3}{4}$

22. $\frac{1}{2p} + \frac{3}{4p} < \frac{1}{2}$

23. $\frac{b-4}{b-2} = \frac{b-2}{b+2} + \frac{1}{b-2}$

24. $\frac{4n^2}{n^2-9} - \frac{2n}{n+3} = \frac{3}{n-3}$

25. $\frac{1}{d+4} = \frac{2}{d^2+3d-4} - \frac{1}{1-d}$

26. $\frac{2}{y+2} - \frac{y}{2-y} = \frac{y^2+4}{y^2-4}$

27. $\frac{3}{b^2+5b+6} + \frac{b-1}{b+2} = \frac{7}{b+3}$

28. $\frac{1}{n-2} = \frac{2n+1}{n^2+2n-8} + \frac{2}{n+4}$

29. $\frac{2q}{2q+3} - \frac{2q}{2q-3} = 1$

30. $\frac{4}{z-2} - \frac{z+6}{z+1} = 1$

31. NUMBER THEORY The ratio of 8 less than a number to 28 more than that number is 2 to 5. What is the number?

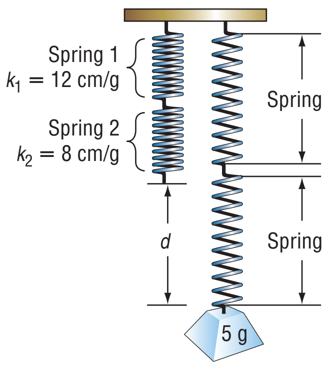
32. NUMBER THEORY The sum of a number and 8 times its reciprocal is 6. Find the number(s).

33. ACTIVITIES The band has 30 more members than the school chorale. If each group had 10 more members, the ratio of their membership would be 3:2. How many members are in each group?

PHYSICS **For Exercises 34 and 35, use the following information.**
The distance a spring stretches is related to the mass attached to the spring. This is represented by $d = km$, where d is the distance, m is the mass, and k is the spring constant. When two springs with spring constants k_1 and k_2 are attached in a series, the resulting spring constant k is found by the equation $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$.

34. If one spring with constant of 12 centimeters per gram is attached in a series with another spring with constant of 8 centimeters per gram, find the resultant spring constant.

35. If a 5-gram object is hung from the series of springs, how far will the springs stretch?



36. CYCLING On a particular day, the wind added 3 kilometers per hour to Alfonso's rate when he was cycling with the wind and subtracted 3 kilometers per hour from his rate on his return trip. Alfonso found that in the same amount of time he could cycle 36 kilometers with the wind, he could go only 24 kilometers against the wind. What is his normal bicycling speed with no wind?

Career Choices

Chemist

Many chemists work for manufacturers developing products or doing quality control to ensure the products meet industry and government standards.

Online Research For information about a career as a chemist, visit: www.algebra2.com/careers

37. **CHEMISTRY** Kiara adds an 80% acid solution to 5 milliliters of solution that is 20% acid. The function that represents the percent of acid in the resulting solution is $f(x) = \frac{5(0.20) + x(0.80)}{5 + x}$, where x is the amount of 80% solution added. How much 80% solution should be added to create a solution that is 50% acid?

STATISTICS **For Exercises 38 and 39, use the following information.**

A number x is the *harmonic mean* of y and z if $\frac{1}{x}$ is the average of $\frac{1}{y}$ and $\frac{1}{z}$.

38. Find y if $x = 8$ and $z = 20$.

39. Find x if $y = 5$ and $z = 8$.

40. **CRITICAL THINKING** Solve for a if $\frac{1}{a} - \frac{1}{b} = c$.

41. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are rational equations used to solve problems involving unit price?

Include the following in your answer:

- an explanation of how to solve $\frac{500 + 5x}{x} = 6$, and
- the reason why the actual price per minute could never be 5¢.

Standardized Test Practice

42. If $T = \frac{4st}{s - t}$, what is the value of s when $t = 5$ and $T = 40$?

(A) 20 (B) 10 (C) 5 (D) 2

43. Amanda wanted to determine the average of her 6 test scores. She added the scores correctly to get T, but divided by 7 instead of 6. Her average was 12 less than the actual average. Which equation could be used to determine the value of T?

(A) $6T + 12 = 7T$

(B) $\frac{T}{7} = \frac{T - 12}{6}$

(C) $\frac{T}{7} + 12 = \frac{T}{6}$

(D) $\frac{T}{6} = \frac{T - 12}{7}$

Maintain Your Skills

Mixed Review

Identify the type of function represented by each equation. Then graph the equation. *(Lesson 9-5)*

44. $y = 2x^2 + 1$

45. $y = 2\sqrt{x}$

46. $y = 0.8x$

47. If y varies inversely as x and $y = 24$ when $x = 9$, find y when $x = 6$. *(Lesson 9-4)*

48. If y varies directly as x and $y = 9$ when $x = 4$, find y when $x = 15$. *(Lesson 9-4)*

Find the distance between each pair of points with the given coordinates. *(Lesson 8-1)*

49. $(-5, 7), (9, -11)$

50. $(3, 5), (7, 3)$

51. $(-1, 3), (-5, -8)$

Solve each inequality. *(Lesson 6-7)*

52. $(x + 11)(x - 3) > 0$

53. $x^2 - 4x \leq 0$

54. $2b^2 - b < 6$

Graphing Calculator Investigation

A Follow-Up of Lesson 9-6

Solving Rational Equations by Graphing

You can use a graphing calculator to solve rational equations. You need to graph both sides of the equation and locate the point(s) of intersection. You can also use a graphing calculator to confirm solutions that you have found algebraically.

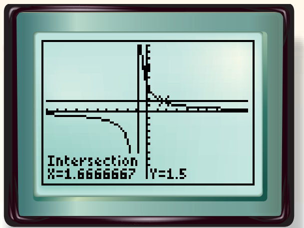
Example

Use a graphing calculator to solve $\frac{4}{x+1} = \frac{3}{2}$.

- First, rewrite as two functions, $y_1 = \frac{4}{x+1}$ and $y_2 = \frac{3}{2}$.
- Next, graph the two functions on your calculator.

  KEYSTROKES: [Y=] 4 [÷] [(] [X,T,θ,n] [+] 1 [)] [▼] 3 [÷] 2 [ZOOM] 6

  Notice that because the calculator is in connected mode, a vertical line is shown connecting the two branches of the hyperbola. This line is not part of the graph.



[−10, 10] scl: 1 by [−10, 10] scl: 1

- Next, locate the point(s) of intersection.

  KEYSTROKES: [2nd] [CALC] 5

  Select one graph and press [ENTER]. Select the other graph, press [ENTER], and press [ENTER] again. The solution is $1\frac{2}{3}$. Check this solution by substitution.

Exercises

Use a graphing calculator to solve each equation.

1. $\frac{1}{x} + \frac{1}{2} = \frac{2}{x}$

2. $\frac{1}{x-4} = \frac{2}{x-2}$

3. $\frac{4}{x} = \frac{6}{x^2}$

4. $\frac{1}{1-x} = 1 - \frac{x}{x-1}$

5. $\frac{1}{x+4} = \frac{2}{x^2+3x-4} - \frac{1}{1-x}$

6. $\frac{1}{x-1} + \frac{1}{x+2} = \frac{1}{2}$

Solve each equation algebraically. Then, confirm your solution(s) using a graphing calculator.

7. $\frac{3}{x} + \frac{7}{x} = 9$

8. $\frac{1}{x-1} + \frac{2}{x} = 0$

9. $1 + \frac{5}{x-1} = \frac{7}{6}$

10. $\frac{1}{x^2-1} = \frac{2}{x^2+x-2}$

11. $\frac{6}{x^2+2x} - \frac{x+1}{x+2} = \frac{2}{x}$

12. $\frac{3}{x^2+5x+6} + \frac{x-1}{x+2} = \frac{7}{x+3}$

www.algebra2.com/other_calculator_keystrokes

Chapter 9 Study Guide and Review

Vocabulary and Concept Check

asymptote (p. 485)	direct variation (p. 492)	rational equation (p. 505)
complex fraction (p. 475)	inverse variation (p. 493)	rational expression (p. 472)
constant of variation (p. 492)	joint variation (p. 493)	rational function (p. 485)
continuity (p. 485)	point discontinuity (p. 485)	rational inequality (p. 508)

State whether each sentence is *true* or *false*. If false, replace the underlined word or number to make a true sentence.

1. The equation $y = \frac{x^2 - 1}{x + 1}$ has a(n) <u>*asymptote*</u> at $x = -1$.
2. The equation $y = 3x$ is an example of a <u>*direct*</u> variation equation.
3. The equation $y = \frac{x^2}{x + 1}$ is a(n) <u>*polynomial*</u> equation.
4. The graph of $y = \frac{4}{x - 4}$ has a(n) <u>*variation*</u> at $x = 4$.
5. The equation $b = \frac{2}{a}$ is a(n) <u>*inverse*</u> variation equation.
6. On the graph of $y = \frac{x - 5}{x + 2}$, there is a break in continuity at $x = \underline{2}$.

Lesson-by-Lesson Review

9-1 Multiplying and Dividing Rational Expressions

See pages 472–478.

Concept Summary

- Multiplying and dividing rational expressions is similar to multiplying and dividing fractions.

Examples

1 Simplify $\frac{3x}{2y} \cdot \frac{8y^3}{6x^2}$.

$$\frac{3x}{2y} \cdot \frac{8y^3}{6x^2} = \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot \overset{1}{\cancel{y}} \cdot y \cdot y}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{y}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{x}} \cdot x}$$

$$= \frac{2y^2}{x}$$

2 Simplify $\frac{p^2 + 7p}{3p} \div \frac{49 - p^2}{3p - 21}$.

$$\frac{p^2 + 7p}{3p} \div \frac{49 - p^2}{3p - 21} = \frac{p^2 + 7p}{3p} \cdot \frac{3p - 21}{49 - p^2}$$

$$= \frac{\overset{1}{\cancel{p}}\overset{1}{\cancel{(7 + p)}}}{\underset{1}{\cancel{3}}\underset{1}{\cancel{p}}} \cdot \frac{\overset{-1}{\cancel{-3}}\overset{1}{\cancel{(7 - p)}}}{\underset{1}{\cancel{(7 + p)}}\underset{1}{\cancel{(7 - p)}}}$$

$$= -1$$

Exercises **Simplify each expression.** *See Examples 4–7 on pages 474 and 475.*

7. $\frac{-4ab}{21c} \cdot \frac{14c^2}{22a^2}$
8. $\frac{a^2 - b^2}{6b} \div \frac{a + b}{36b^2}$
9. $\frac{y^2 - y - 12}{y + 2} \div \frac{y - 4}{y^2 - 4y - 12}$
10. $\dfrac{\frac{x^2 + 7x + 10}{x + 2}}{\frac{x^2 + 2x - 15}{x + 2}}$
11. $\dfrac{\frac{1}{n^2 - 6n + 9}}{\frac{n + 3}{2n^2 - 18}}$
12. $\frac{x^2 + 3x - 10}{x^2 + 8x + 15} \cdot \frac{x^2 + 5x + 6}{x^2 + 4x + 4}$

9-2 Adding and Subtracting Rational Expressions

See pages 479–484.

Concept Summary

- To add or subtract rational expressions, find a common denominator.
- To simplify complex fractions, simplify the numerator and the denominator separately, and then simplify the resulting expression.

Example Simplify $\frac{14}{x+y} - \frac{9x}{x^2 - y^2}$.

$\frac{14}{x+y} - \frac{9x}{x^2-y^2} = \frac{14}{x+y} - \frac{9x}{(x+y)(x-y)}$ Factor the denominators.

$= \frac{14(x-y)}{(x+y)(x-y)} - \frac{9x}{(x+y)(x-y)}$ The LCD is $(x+y)(x-y)$.

$= \frac{14(x-y) - 9x}{(x+y)(x-y)}$ Subtract the numerators.

$= \frac{14x - 14y - 9x}{(x+y)(x-y)}$ Distributive Property

$= \frac{5x - 14y}{(x+y)(x-y)}$ Simplify.

Exercises Simplify each expression. *See Examples 3 and 4 on page 480.*

13. $\frac{x+2}{x-5} + 6$
14. $\frac{x-1}{x^2-1} + \frac{2}{5x+5}$
15. $\frac{7}{y} - \frac{2}{3y}$
16. $\frac{7}{y-2} - \frac{11}{2-y}$
17. $\frac{3}{4b} - \frac{2}{5b} - \frac{1}{2b}$
18. $\frac{m+3}{m^2-6m+9} - \frac{8m-24}{9-m^2}$

9-3 Graphing Rational Functions

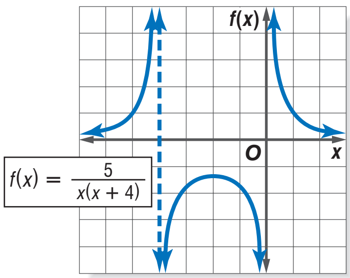
See pages 485–490.

Concept Summary

- Functions are undefined at any x value where the denominator is zero.
- An asymptote is a line that the graph of the function approaches, but never crosses.

Example Graph $f(x) = \frac{5}{x(x+4)}$.

The function is undefined for $x = 0$ and $x = -4$. Since $\frac{5}{x(x+4)}$ is in simplest form, $x = 0$ and $x = -4$ are vertical asymptotes. Draw the two asymptotes and sketch the graph.



Exercises Graph each rational function. *See Examples 2–4 on pages 486–488.*

19. $f(x) = \frac{4}{x-2}$
20. $f(x) = \frac{x}{x+3}$
21. $f(x) = \frac{2}{x}$
22. $f(x) = \frac{x-4}{x+3}$
23. $f(x) = \frac{5}{(x+1)(x-3)}$
24. $f(x) = \frac{x^2+2x+1}{x+1}$

9-4 Direct, Joint, and Inverse Variation

See pages 492–498.

Concept Summary

- Direct Variation: There is a nonzero constant k such that $y = kx$.
- Joint Variation: There is a number k such that $y = kxz$, where $x \neq 0$ and $z \neq 0$.
- Inverse Variation: There is a nonzero constant k such that $xy = k$ or $y = \frac{k}{x}$.

Example **If y varies inversely as x and $x = 14$ when $y = -6$, find x when $y = -11$.**

$\frac{x_1}{y_2} = \frac{x_2}{y_1}$ Inverse variation

$\frac{14}{-11} = \frac{x_2}{-6}$ $x_1 = 14, y_1 = -6, y_2 = -11$

$14(-6) = -11(x_2)$ Cross multiply.

$-84 = -11x_2$ Simplify.

$7\frac{7}{11} = x_2$ When $y = -11$, the value of x is $7\frac{7}{11}$.

Exercises **Find each value.** *See Examples 1–3 on pages 493 and 494.*

25. If y varies directly as x and $y = 21$ when $x = 7$, find x when $y = -5$.

26. If y varies inversely as x and $y = 9$ when $x = 2.5$, find y when $x = -0.6$.

27. If y varies inversely as x and $x = 28$ when $y = 18$, find x when $y = 63$.

28. If y varies directly as x and $x = 28$ when $y = 18$, find x when $y = 63$.

29. If y varies jointly as x and z and $x = 2$ and $z = 4$ when $y = 16$, find y when $x = 5$ and $z = 8$.

9-5 Classes of Functions

See pages 499–504.

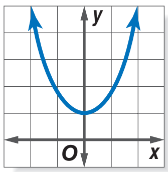
Concept Summary

The following is a list of special functions.

- constant function
- direct variation function
- identity function
- greatest integer function
- absolute value function
- quadratic function
- square root function
- rational function
- inverse variation function

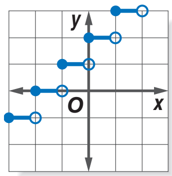
Examples **Identify the type of function represented by each graph.**

1



The graph has a parabolic shape, therefore it is a quadratic function.

2

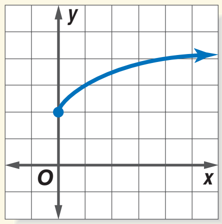


The graph has a stair-step pattern, therefore it is a greatest integer function.

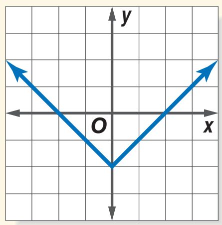
• Extra Practice, see pages 847–849.
• Mixed Problem Solving, see page 870.

Exercises **Identify the type of function represented by each graph.**
See Example 1 on page 500.

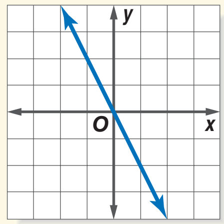
30.



31.



32.



9-6 Solving Rational Equations and Inequalities

See pages 505–511.

Concept Summary

- Eliminate fractions in rational equations by multiplying each side of the equation by the LCD.
- Possible solutions to a rational equation must exclude values that result in zero in the denominator.
- To solve rational inequalities, find the excluded values, solve the related equation, and use these values to divide a number line into regions. Then test a value in each region to determine which regions satisfy the original inequality.

Example **Solve** $\frac{1}{x-1} + \frac{2}{x} = 0$.

The LCD is $x(x - 1)$.

$$\frac{1}{x-1} + \frac{2}{x} = 0 \quad \text{Original equation}$$

$$x(x-1)\left(\frac{1}{x-1} + \frac{2}{x}\right) = x(x-1)(0) \quad \text{Multiply each side by } x(x-1).$$

$$x(x-1)\left(\frac{1}{x-1}\right) + x(x-1)\left(\frac{2}{x}\right) = x(x-1)(0) \quad \text{Distributive Property}$$

$$1(x) + 2(x-1) = 0 \quad \text{Simplify.}$$

$$x + 2x - 2 = 0 \quad \text{Distributive Property}$$

$$3x - 2 = 0 \quad \text{Simplify.}$$

$$3x = 2 \quad \text{Add 2 to each side.}$$

$$x = \frac{2}{3} \quad \text{Divide each side by 3.}$$

The solution is $\frac{2}{3}$.

Exercises **Solve each equation or inequality. Check your solutions.**
See Examples 1, 2, and 5 on pages 505, 506, 508, and 509.

33. $\frac{3}{y} + \frac{7}{y} = 9$

34. $1 + \frac{5}{y-1} = \frac{7}{6}$

35. $\frac{3x+2}{4} = \frac{9}{4} - \frac{3-2x}{6}$

36. $\frac{1}{r^2-1} = \frac{2}{r^2+r-2}$

37. $\frac{x}{x^2-1} + \frac{2}{x+1} = 1 + \frac{1}{2x-2}$

38. $\frac{1}{3b} - \frac{3}{4b} > \frac{1}{6}$

CONTENTS

Chapter 9 Practice Test

Vocabulary and Concepts

Match each example with the correct term.

1. $y = 4xz$
2. $y = 5x$
3. $y = \frac{7}{x}$

a. inverse variation equation
b. direct variation equation
c. joint variation equation

Skills and Applications

Simplify each expression.

4. $\frac{a^2 - ab}{3a} \div \frac{a - b}{15b^2}$

5. $\frac{x^2 - y^2}{y^2} \cdot \frac{y^3}{y - x}$

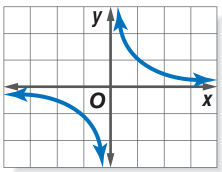
6. $\frac{x^2 - 2x + 1}{y - 5} \div \frac{x - 1}{y^2 - 25}$

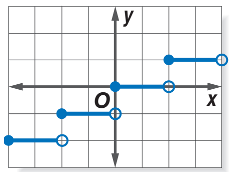
7. $\dfrac{\frac{x^2 - 1}{x^2 - 3x - 10}}{\frac{x^2 + 3x + 2}{x^2 - 12x + 35}}$

8. $\frac{x - 2}{x - 1} + \frac{6}{7x - 7}$

9. $\frac{x}{x^2 - 9} + \frac{1}{2x + 6}$

Identify the type of function represented by each graph.

10.


11.


Graph each rational function.

12. $f(x) = \frac{-4}{x - 3}$

13. $f(x) = \frac{2}{(x - 2)(x + 1)}$

Solve each equation or inequality.

14. $\frac{2}{x - 1} = 4 - \frac{x}{x - 1}$

15. $\frac{9}{28} + \frac{3}{z + 2} = \frac{3}{4}$

16. $5 + \frac{3}{t} > -\frac{2}{t}$

17. $x + \frac{12}{x} - 8 = 0$

18. $\frac{5}{6} - \frac{2m}{2m + 3} = \frac{19}{6}$

19. $\frac{x - 3}{2x} = \frac{x - 2}{2x + 1} - \frac{1}{2}$

20. If y varies inversely as x and $y = 9$ when $x = -\frac{2}{3}$, find x when $y = -7$.

21. If g varies directly as w and $g = 10$ when $w = -3$, find w when $g = 4$.

22. Suppose y varies jointly as x and z. If $x = 10$ when $y = 250$ and $z = 5$, find x when $y = 2.5$ and $z = 4.5$.

23. **AUTO MAINTENANCE** When air is pumped into a tire, the pressure required varies inversely as the volume of the air. If the pressure is 30 pounds per square inch when the volume is 140 cubic inches, find the pressure when the volume is 100 cubic inches.

24. **ELECTRICITY** The current I in a circuit varies inversely with the resistance R.
    a. Use the table at the right to write an equation relating the current and the resistance.
    b. What is the constant of variation?

I	0.5	1.0	1.5	2.0	2.5	3.0	5.0
R	12.0	6.0	4.0	3.0	2.4	2.0	1.2

25. **STANDARDIZED TEST PRACTICE** If $m = \frac{1}{x}$, $n = 7m$, $p = \frac{1}{n}$, $q = 14p$, and $r = \frac{1}{\frac{1}{2}q}$, find x.

(A) r (B) q (C) p (D) $\frac{1}{r}$ (E) $\frac{1}{q}$

www.algebra2.com/chapter_test

Chapter 9 Standardized Test Practice

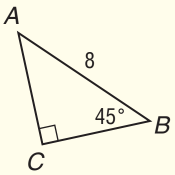
Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Best Bikes has 5000 bikes in stock on May 1. By the end of May, 40 percent of the bikes have been sold. By the end of June, 40 percent of the remaining bikes have been sold. How many bikes remain unsold?

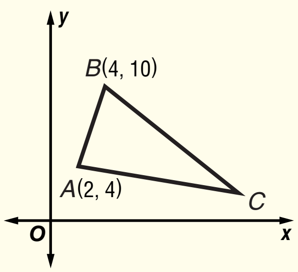
Ⓐ 1000 Ⓑ 1200
Ⓒ 1800 Ⓓ 2000

2. In $\triangle ABC$, if AB is equal to 8, then BC is equal to



Ⓐ $\frac{\sqrt{2}}{8}$. Ⓑ 4.
Ⓒ $4\sqrt{2}$. Ⓓ 8.

3. In the figure, the slope of $\overline{AC}$ is $-\frac{1}{3}$ and $m\angle C = 30°$. What is the length of $\overline{BC}$?



Ⓐ $\sqrt{10}$ Ⓑ $2\sqrt{10}$
Ⓒ $3\sqrt{10}$ Ⓓ $4\sqrt{10}$

4. Given that $-|2 - 4k| = -14$, which of the following could be k?

Ⓐ 5 Ⓑ 4
Ⓒ 3 Ⓓ 2

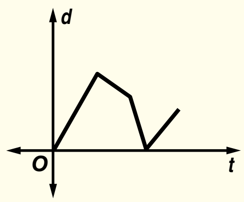
5. In a hardware store, n nails cost c cents. Which of the following expresses the cost of k nails?

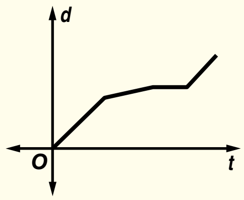
Ⓐ nck Ⓑ $\frac{kc}{n}$
Ⓒ $n + \frac{k}{c}$ Ⓓ $n + \frac{c}{n}$

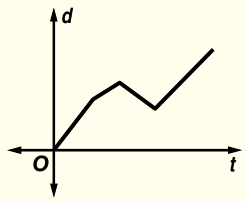
6. If $5w + 3 \leq w - 9$, then

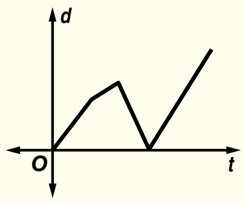
Ⓐ $w \leq 3$. Ⓑ $w \geq 3$.
Ⓒ $w \leq 12$. Ⓓ $w \leq -3$.

7. The graphs show a driver's distance d from a designated point as a function of time t. The driver passed the designated point at 60 mph and continued at that speed for 2 hours. Then she slowed to 50 mph for 1 hour. She stopped for gas and lunch for 1 hour and then drove at 60 mph for 1 hour. Which graph best represents this trip?

Ⓐ


Ⓑ


Ⓒ


Ⓓ


8. Which equation has roots of $-2n$, $2n$, and 2?

Ⓐ $2x^2 - 8n^2 = 0$
Ⓑ $8n^2 - 2x^2 = 0$
Ⓒ $x^3 - 2x^2 - 4n^2x - 8n^2 = 0$
Ⓓ $x^3 - 2x^2 - 4n^2x + 8n^2 = 0$

9. What point is on the graph of $y - x^2 = 2$ and has a y-coordinate of 5?

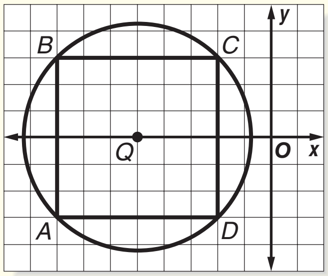
Ⓐ $(-\sqrt{3}, 5)$ Ⓑ $(\sqrt{7}, 5)$
Ⓒ $(5, \sqrt{3})$ Ⓓ $(3, 5)$

CONTENTS

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. In the figure, what is the equation of the circle *Q* that is circumscribed around the square *ABCD*?



11. Find one possible value for *k* such that *k* is an integer between 20 and 40 that has a remainder of 2 when it is divided by 3 and that has a remainder of 2 when divided by 4.

12. The coordinates of the vertices of a triangle are $(2, -4)$, $(10, -4)$, and (a, b). If the area of the triangle is 36 square units, what is a possible value for b?

13. If $(x + 2)(x - 3) = 6$, what is a possible value of x?

14. If the average of five consecutive even integers is 76, what is the greatest of these integers?

15. In May, Hank's Camping Supply Store sold 45 tents. In June, it sold 90 tents. What is the percent increase in the number of tents sold?

16. If $2^{n-4} = 64$, what is the value of n?

17. If $xy = 5$ and $x^2 + y^2 = 20$, what is the value of $(x + y)^2$?

18. If $\frac{2}{a} - \frac{8}{a^2} = \frac{-8}{a^3}$, then what is the value of a?

19. If $\sqrt[x]{80} = 2\sqrt[x]{5}$, what is the value of x?

20. What is the y-intercept of the graph of $3x + 2 = 4y - 6$?

www.algebra2.com/standardized_test

Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

- **(A) the quantity in Column A is greater,**
- **(B) the quantity in Column B is greater,**
- **(C) the two quantities are equal, or**
- **(D) the relationship cannot be determined from the information given.**

	Column A	Column B
21.	the number of distinct prime factors of 105	the number of distinct prime factors of 189

22. $\frac{x}{y} = \frac{3}{7}$

Column A	Column B
x	y

23.

Column A	Column B
t	$3t$

24. $0 < x < 1$

Column A	Column B
x^2	x^3

25. $0 < x < 1$

Column A	Column B
x	$\sqrt{x}$

The Princeton Review Test-Taking Tip

Questions 22–25 In quantitative comparison questions that involve variables, make sure you consider all of the possible values of the variables before you make a comparison. Consider positive and negative integers, positive and negative fractions, and 0.